Comparison of Actual Delivered Density and Fire Suppression Effectiveness of Standard and Conventional Sprinklers in Rack-Storage Fires

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ABSTRACT

The Actual Delivered Density (ADD) of a standard sprinkler and a conventional sprinkler for rack-storage fires has been studied experimentally using an ADD apparatus equipped with a fire plume simulator. The variables examined included sprinkler discharge rates (76-212 L/min), heat release rates of the fire (0, 500, 1000, 1500 and 2000 kW), ceiling clearances (3.05, 4.57 and 6.10 m), and ignition locations (centered below one sprinkler and below four sprinklers). The sprinkler spacing was maintained at 3.05 m for all the tests. The results of the measurements indicate that the ADD decreased as the ceiling clearance is increased from 3.05 m to 6.10 m. The ADD data have been correlated satisfactorily using two parameters. One parameter corresponds to a drop size ratio and the other is the ratio of the spray momentum to the momentum of the fire plume. It is demonstrated that the ADD can be used to interpret the performance of the standard and conventional sprinklers in the large-scale rack storage fire tests conducted.


1. INTRODUCTION

The automatic sprinkler has been used to protect against fires for over one hundred years. When first introduced in 1878, the sprinkler was designed to project approximately half of the water to the ceiling and half toward the floor. This design philosophy had not been changed until Factory Mutual (FM) introduced the “spray” sprinkler in the 1950’s in recognition of the important role of water distribution over the fire on the floor. The deflector of the spray sprinkler was designed to project all the water downward toward the fire on the floor. After several years of successful demonstration of its fire protection effectiveness, the spray sprinkler was accepted in 1953 by the National Fire Protection Association as the “standard” sprinkler in the United States and the early generation sprinklers were gradually phased out in the United States. However, the early generation sprinklers continue to find application in Europe, where they are referred to as “conventional” sprinklers. The conventional sprinkler is still preferred over the standard sprinkler in Europe. Standard sprinklers have a nominal diameter of 12.7 mm and a K factor (discharge coefficient) of 8 L/min-kPa^{1/2}.

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For the past four decades, since the introduction of the standard sprinkler, industrial
occupancies have undergone dramatic changes in manufacturing and storage practices,
such as the introduction of new synthetic materials and use of forklift trucks that allow
high-rack storage arrangement. Fires in this rack storage environment are characterized
by extremely fast fire growth, high heat release rate, and high plume velocity, and have
challenged the standard sprinkler to its limit of effectiveness. In response to these
challenges, FM initiated a series of research programs to study the principles of sprinkler
fire protection in an effort to provide a sound technical basis for sprinkler system design.
In the 1970’s, an extensive laboratory study was undertaken by Heskestad[1] to
investigate the interaction of sprinkler sprays with buoyant plumes. Sprinkler
performance parameters that govern the performance of a sprinkler system were
identified and correlated based on dimensional analysis of pertinent variables.

The performance-parameter concept was further developed and tested in the 1980’s when
FM embarked on the Early Suppression Fast Response (ESFR) Research Program[2]. In
the program, a systematic approach was developed to predict sprinkler fire suppression
effectiveness based on an important sprinkler performance parameter called the Actual
Delivered Density (ADD), and a characteristic of the fuel array extinguishability referred
to as Required Delivered Density (RDD). The ADD is the water flux delivered by the
sprinkler(s) to the top surface of a burning fuel array after penetrating through the fire
plume. The RDD is the water flux that must be delivered to the top surface of a burning
fuel array in order to achieve fire suppression. Fire suppression is expected to occur when
the ADD is greater than the RDD.

An experimental apparatus was designed and constructed to study the Actual Delivered
Density (ADD) of sprinklers for large-scale rack storage fires[3]. This paper presents the
ADD data for a standard and a conventional sprinkler, using the experimental apparatus.
Correlations of ADD data that are affected by water discharge rate (or pressure), fire size,
ceiling clearance and ignition location have been developed. Also, the ADD/RDD
concept has been applied to interpret results of full-scale fire tests[4] in which protection
was provided by the selected standard and conventional sprinklers.

2. MEASUREMENT SETUPS AND PROCEDURES

Measurements were conducted using an ADD apparatus to study the ADD of a standard
upright sprinkler (Grinnell F950 Spray 1/2 in.) and a conventional upright sprinkler
(Grinnell F950 Conventional 1/2 in.). The two sprinklers use the same frame but with
different deflectors. The K factor of the two sprinklers is 8 L/min⋅kPa$^{1/2}$. To examine the
effect of ceiling clearance on the water penetration in the presence of a fire, the
measurements were performed for three ceiling clearances, corresponding to 3.05, 4.57
and 6.10 m between the ceiling and the top of the water collection assembly of the ADD
apparatus. For each ceiling clearance, the measurements were performed for two
sprinkler-fire arrangements in which the fire was directly below a sprinkler or centered
below four sprinklers. The fire heat release rates employed for the measurements were 0,
500, 1000, 1500 and 2000 kW, while the sprinkler discharge rates were 76, 95, 114, 140,
170 and 212 L/min.

The measurements were conducted in Bldg. 18 of the FM Norwood facility under a 9.93 m x
11.23 m suspended ceiling. The ceiling could be raised or lowered to provide several floor-to-
ceiling clearances. Overall dimensions of the test site were 12.2 m x 18.3 m x 10.1 m high.
Figure 1 shows a plan view of the ADD apparatus. It consists of two parts: 1) a fire plume simulator and 2) a water collection pan assembly. The fire plume simulator consists of nine spray nozzles using heptane as fuel. Eight nozzles are equally spaced on a 1.22-m diameter circle with one nozzle located at the center of the circle above a 102-mm-diameter air-deflecting plate. A 203-mm diameter duct is placed directly underneath the plate, supplying standard air at 18.4 m³/min. A detailed description of the apparatus can be found in Reference [3].

Different fire sizes are obtained by changing the spray nozzle size, nozzle angle with respect to the horizontal plane, and the fuel supply pressure. The settings of these variables for each selected fire size are listed in Table 1 of Reference [3]. The operating conditions shown in the table were determined so that the fire plumes generated by the heptane spray fire would simulate the plumes of a 6.1 m - high rack storage fire of the FMRC Class II Commodity (metal-lined double triwall cartons on pallets). This was accomplished by matching the temperature and velocity measurements of the heptane-spray fire plumes at three elevations with those of the large-scale rack storage fire at the same heat release rate[3,5].

The water collection assembly consists of a total of twenty pans that are assembled to simulate the top surface of a rack storage fuel array, two-pallet loads wide by two-pallet loads deep, including the flue space between the commodities. The sixteen square pans represent the top surfaces of the commodities; the four rectangular pans represent the flue spaces. The undersides of the water collection pans are cooled by water spray nozzles in order to keep the pan temperature from significantly increasing due to flame radiation. The water collected in each pan is channeled to a container via a PVC hose. Each container is equipped with a pressure sensor (SenSym model SCX01) to monitor the instantaneous water level while water is being collected. The water collection rate in each container is then calculated based on the measured water level data.

Open sprinklers were installed on sprinkler pipes of nominal 2-in. diameter under the suspended ceiling via threaded bushings. The centerlines of the sprinkler pipes were 0.32 m below the ceiling. The two deflector-supporting arms of each sprinkler were aligned with the sprinkler pipe. Water flow to the sprinkler(s) could be turned on or off rapidly, by a pneumatic ball valve situated at the inlet of the sprinkler pipe system. The total water flow to the open sprinkler(s) was controlled with a flow control system, which included a paddle-wheel flow meter (Omega Series FP-5200 Flow Sensor), an electrically operated cage valve (Honeywell 8107 valve & M740 actuator) and a flow controller (Honeywell UDC 3000). To monitor the water discharge pressure, a pressure tap was installed at the downstream end of each sprinkler pipe and connected via nominal 1/4-in. tubing to a pressure transducer (Setra model 205-2).

Before each measurement, water was discharged through the open sprinkler(s), and the pressure was adjusted to the designated value using the flow control system. The water flow rate was also checked to ensure consistency with the discharge pressure. The water flow to the sprinkler(s) was then shut off and all the containers receiving water from the collection pans were emptied. All data (water flow and pressure, and container water levels) were recorded by an HP data acquisition system with a sampling rate of one scan per second. Prior to ignition, the computer collected pretest data for 30 seconds. The heptane spray was then ignited and allowed to burn for 40 seconds before actuation of the pneumatic valve to start the water flow. After water started to discharge from the
sprinkler(s), the test continued for 5 additional minutes. Then the water flow to the sprinkler(s) and the fuel to the simulator were shut off. Smoke inside the building was cleared before the next measurement. During each test, all doors to the test site were closed, whereas the two hatches in the roof and the windows in the building wall immediately below the roof were open.

![Plan view of the ADD apparatus.](image)

**Figure 1.** Plan view of the ADD apparatus.

<table>
<thead>
<tr>
<th>Ceiling Clearance (m)</th>
<th>ADD (lpm/m²)</th>
<th>Convective Heat Release Rate (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.05</td>
<td>Conv. Spr.</td>
<td>0</td>
</tr>
<tr>
<td>4.57</td>
<td>Spray Spr.</td>
<td>5</td>
</tr>
<tr>
<td>6.10</td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>

**Figure 2.** ADD variation with ceiling clearance for Grinnell F950 sprinklers. Sprinkler discharge rate: 140 l/min per sprinkler.

![ADD variation with ceiling clearance for Grinnell F950 sprinklers.](image)

a) Fire centered under one sprinkler  
b) Fire centered below four sprinklers
3. MEASUREMENT RESULTS AND DATA ANALYSIS

The Actual Delivered Density (ADD) is calculated based on the individual water collection rate measurements. The ADD is defined as the average water density (flux) over the area of the entire 20 water-collection pans; the area covered by these pans corresponds to the top surface of the ignition array, including flue spaces, in a large-scale rack storage fire test.

Figures 2a and 2b present comparisons of the ADDs of Grinnell F950 Spray and Conventional sprinklers at a sprinkler discharge rate of 140 L/min for the cases when the fire is centered below one sprinkler and four sprinklers, respectively. The ADD is plotted against the convective heat release rate of the fire and decreases with increasing heat release rates. For each sprinkler, the ADDs have been measured for three ceiling clearances (3.05, 4.57 and 6.10 m), and are presented in Fig. 2a and 2b. In general, the ADD decreases with increasing ceiling clearance. For the case with the fire centered below one sprinkler, the ADD of the standard sprinkler is greater than that of the conventional sprinkler at heat release rates equal to or greater than 1000 kW; however, at zero heat release rate (no fire condition), the ADD of the conventional sprinkler is greater than that of the standard sprinkler. For the case with the fire centered below four sprinklers, the ADD of the spray sprinkler is always considerably higher than that of the conventional sprinkler.

The following functional form is used to correlate the water penetration to the ignition area:

\[
\text{Pe} \equiv \frac{\text{ADD}}{\text{LAD}} \propto \xi^n f(\lambda),
\]

where

\[
\frac{d_m}{d_c} \propto \xi = \left( \frac{g^2 \rho_w \sigma}{\rho_\infty^2 \mu_c} \right)^{1/3} \frac{D_{or}^2}{u_p \bar{Q}_w^{2/3}},
\]

\[
\frac{\dot{m}_s}{\dot{m}_p} \propto \lambda = \frac{\rho_w \bar{Q}_w u_w}{\dot{m}_p},
\]

and LAD is the water flux in the absence of a fire. The parameter \(\xi\) corresponds to a drop size ratio – the ratio of the drop size of the sprinkler spray to a minimum drop size below which drops would be stopped or deflected by the upward gas flow of the fire plume. The parameter \(\lambda\) corresponds to the ratio of the spray momentum flow to that of the fire plume. Expressions used to evaluate \(\xi\) and \(\lambda\) are presented in Appendix A.

The function \(f(\lambda)\) is determined by plotting \(\text{Pe}/\xi^n\) versus \(\lambda\) for different values of the exponent \(n\). Fig. 3a presents the results which \(n\) provides the best fits of the experimental data for the single-sprinkler testing. The values of \(\xi\) and \(\lambda\) are calculated based on the plume velocity and momentum flow at the ceiling elevation. It is seen that, with a single value of \(n = 0.8\), the data for the standard sprinkler correlate well according to the functional relationship presented in Eq. (1). With \(n = 1.2\), the data for the conventional sprinkler also correlate satisfactorily with Eq. (1).
Figure 3. Correlation of water penetration for Grinnell F950 (1/2 in.) sprinklers.
The data for the four-sprinkler arrangement are also analyzed using Eq. (1), but with \( u_p \) in \( \xi \) being set to the maximum velocity of the fire plume because under the arrangement, droplets enter the fire plume from the side and the maximum plume velocity is regarded as the characteristic plume velocity. The results of the analysis indicate that when the fire is centered below four sprinklers, the water penetration to the ignition area also correlates according to Eq. (1), as shown in Fig. 3b. For the conventional sprinkler, the rate of increase of the function \( f(\lambda) \) with \( \lambda \) is greater than that of the standard sprinklers. For the arrangement of fire centered below four sprinklers, \( f(\lambda) \) of the standard sprinklers tends to approach a constant value when \( \lambda \) decreases below 0.3, indicating a region in which the water penetration depends only on the drop size parameter. This region has also been observed by Heskestad[1] in his experiments involving a non-reactive fire plume simulator.

4. INTERPRETATION OF FULL-SCALE FIRE TEST RESULTS

In this section, the ADD/RDD concept is used to analyze the results of full-scale sprinklered fire tests involving the selected standard and conventional sprinklers used in this study. The full-scale fire tests being analyzed are documented in Reference [4]. The fuel arrays used in the tests were comprised of the FMRC Class II Commodity (double tri-wall cartons lined with sheet metal), which were stored in four tiers of steel racks to a height of 6.10 m under a 9.14 m high ceiling. Protection was provided by sprinklers installed on 3.05 m x 3.05 m spacing and discharging at a rate of 140 L/min per sprinkler.

Table 1 summarizes the fire test results for the standard (Grinnell F950 spray) and conventional (Grinnell F950 conventional) sprinklers. The tests involved two different ignition locations: directly under a sprinkler and centered below four sprinklers. When the ignition was directly under a sprinkler, the fire was neither suppressed by the conventional sprinklers nor by the standard sprinklers as demonstrated in Tests 5 and 6, respectively, resulting in a large number of sprinkler operations. When the ignition was centered below four sprinklers, the standard sprinkler, as shown in Test 10, suppressed the fire with four operating sprinklers, whereas the conventional sprinkler in Test 9 failed to suppress the fire, also resulting in large number of sprinkler actuations. When fire suppression occurred, the damage of the commodity was contained to the pallet loads adjacent to the ignition flue space at the center of the array.

The convective heat release rates at first sprinkler actuation listed in the fourth column of Table 1 were obtained from the measured weight loss data and the correlation between the burning rate and the heat release rate[5].

A series of RDD tests was conducted to determine the RDD for the FMRC Class II and plastic commodities in rack storage configurations[6]. The RDD was determined by uniformly applying water to the top of a rack storage array, which was two-pallet loads wide by two-pallet loads deep at the storage height under investigation. The applied water density (flux) was varied to bracket the RDD for a selected fire size. For example, a pair of RDD tests was conducted for the 6.10 m high rack storage array of the Class II commodity with the convective heat release rate at the beginning of water application equal to 1000 kW. The fire in one test was suppressed with 10.2 L/min/m² application density; but with 8.1 L/min/m² application density in the other test, the fire was out of control. Therefore, the RDD for the 6.10-m array is considered to lie between 8 to 10 L/min/m². The results also show that for the range of the fire sizes investigated, 900 – 1500 kW, the RDD is not sensitive to the fire size at the time of water application.
TABLE 1
FIRE TEST RESULTS OF 6.10 m HIGH FMRC CLASS II COMMODITY [REF 4]

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Ignition Scenario</th>
<th>Sprinkler Modela</th>
<th>Convective Heat Release Rate at First Sprinkler Actuation (kW)</th>
<th>Total Number of Sprinklers Operating</th>
<th>Fire Suppression</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Under 1</td>
<td>F950 (Conv ½ in.)</td>
<td>1260</td>
<td>37</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>Under 1</td>
<td>F950 (Spray ½ in.)</td>
<td>880</td>
<td>36</td>
<td>No</td>
</tr>
<tr>
<td>9</td>
<td>Below 4</td>
<td>F950 (Conv ½ in.)</td>
<td>1330</td>
<td>38</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td>Below 4</td>
<td>F950 (Spray ½ in.)</td>
<td>1090</td>
<td>4</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: Water discharge rate was set at 140 L/min per sprinkler.

*aManufactured by Grinnell Corporation.

The RDD tests were further analyzed by Yu, et al[6], using a global energy balance model. In this model, the heat release rate of the fire during the 240 seconds after initiation of water application is represented by the equation:

$$\dot{Q}(t) = \dot{Q}_o e^{-kt_o}$$  \hspace{1cm} \text{for } t > t_o, \hspace{1cm} (2)$$

where $\dot{Q}$ is the total (convective and radiative) heat release rate after water application, $\dot{Q}_o$ is the total heat release rate at the time of water application, and $t_o$ is the time at water application. The “fire-suppression parameter” $k$ is obtained by regression analysis of the experimental data. It is found that for a given commodity, $k$ is a linear function of the water application rate, varying from negative for fire redevelopment to positive for fire suppression. The ratio $\Delta Q/\Delta Q_o$ which is the cumulative heat release in the 240 s period following water application divided by the product of the total heat release rate at the beginning of water application and 240 s, can be used as an indicator of fire suppression. For the cases where the fire is suppressed, the heat release rate decreases with time after water application and the ratio $\Delta Q/\Delta Q_o$ is less than 1. The ratio, $\Delta Q/\Delta Q_o$, has been correlated with $k$ for both the Class II and Plastic commodities at different storage heights. The correlation can be represented by:

$$\Delta Q/\Delta Q_o = [1 – \exp (240 k)]/240k,$$

which can also be obtained by integrating Eq. (2) from $t_o$ to $t_o + 240$. Setting $\Delta Q/\Delta Q_o = 0.8$, $k$ is determined to be 0.001 from Eq. (3). For the FMRC Class II Commodity stored 6.10 m high, the value of RDD for which $k = 0.001$ is determined to be 9.0 L/min/m²[6].

The ADD for the conventional and the spray sprinklers used in the fire tests are shown in Fig. 4a for ignition centered below four sprinklers, and in Fig. 4b for ignition directly under one sprinkler. The ADD values are presented as a function of the free-burn convective heat release rate of the simulated fire plume. The ADD value anticipated for
each of the full-scale fire tests is indicated by an arrow, which corresponds to the heat release rate at first sprinkler actuation listed in the fourth column of Table 1. The numbers associated with the arrows refer to the test numbers in the first column of Table 1. The horizontal dash line denotes the RDD value, 9.0 L/min/m².

Figure 4. Comparison of ADD with the RDD for large-scale fire tests, using Grinnell F950 sprinkler, ceiling clearance: 3.05m, sprinkler discharge rate: 140 l/min per sprinkler.
The ADD at the fire size of first sprinkler actuation, provided by the four operating standard sprinklers in Test 10, is greater than the RDD by 4.1 L/min/m² as shown in Fig. 4a. The fire in Test 10 was suppressed with four operating sprinklers. In contrast, the ADD provided by the four conventional sprinklers closest to the fire in Test 9 is about the same as the RDD, and the fire in the test was not suppressed. A total of 38 sprinklers operated.

For the two tests in which ignition was directly under one sprinkler, i.e., Tests 5 and 6, suppression did not occur. The ADD for these tests was determined to be substantially smaller than the RDD, as shown in Fig. 4b.

CONCLUSIONS

The Actual Delivered Density (ADD) as affected by the sprinkler discharge rate, heat release rate, ceiling clearance and ignition location, has been studied for a standard sprinkler and a conventional sprinkler, using an ADD apparatus equipped with a fire plume simulator. The ADD decreases with increasing fire heat release rate. For the case with the fire centered below one sprinkler, the ADD of the conventional sprinkler is higher than that of the standard sprinkler at zero heat release rate. For the case with the fire centered below four sprinklers, the ADD of the standard sprinkler is considerably higher than that of the conventional sprinkler. In addition, the ADD decreases with increasing ceiling clearance. Using drop size and momentum parameters, the ADD data obtained for the one-, and four-sprinkler arrangements have been correlated successfully using the functional relationship of Eq. (1).

It has been demonstrated that the ADD/RDD concept can be used to interpret the fire suppression effectiveness of standard and conventional sprinklers in full-scale fire tests.

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NOMENCLATURE

\( A_d \) = frontal area of a water drop  
\( C_d \) = drag coefficient  
\( c_p \) = air specific heat  
\( D_{or} \) = sprinkler orifice diameter  
\( d_c \) = critical droplet diameter  
\( d_{m} \) = droplet mass-median diameter  
\( g \) = acceleration of gravity  
\( H \) = ceiling clearance above the water collection pans  
\( k \) = fire suppression parameter  
\( M_p \) = plume momentum flow  
\( M_s \) = spray momentum flux  
\( M_w \) = water discharge momentum  
\( \Delta Q \) = cumulative heat release  
\( Q \) = total heat release rate  
\( Q_c \) = convective heat release rate  
\( Q_w \) = water discharge volumetric flow rate  
\( t \) = time  
\( t_o \) = time of water application  
\( u_p \) = plume centerline velocity  
\( u_w \) = water discharge velocity, \( 4Q_w/(\pi D_{or}^2) \)  
\( \rho \) = air density  
\( \rho_w \) = water density  
\( \mu \) = air viscosity  
\( \sigma_w \) = surface tension (water/air)
REFERENCES


APPENDIX A
Evaluation of Drop-Size and Momentum Ratios

The drop-size ratio, $d_m/d_c$, involves the mass median droplet diameter of the sprinkler spray, $d_m$, and the critical droplet diameter, $d_c$, below which drops would be stopped or deflected by the upward gas flow of the fire plume. For a given sprinkler, the mass median drop size can be expressed explicitly as:

$$d_m/D_or \propto \left( \rho_w u_w^3 D_or / \sigma \right)^{-1/3}$$

where $D_or$ is the orifice diameter of the sprinkler, and $u_w$ is the water velocity discharged through the orifice[7]. The critical droplet diameter is obtained by equating the force of gravity on the drop to the upward drag force of plume gas when the drop is suspended in the flow, i.e.,

$$1/2 \rho_g u_p^2 C_d A_d = \rho_w g \pi d_c^3 / 6$$

where $C_d$ is the drag coefficient, $A_d$ the frontal area of the drop, and $u_p$ is the velocity of the plume gas flow. The drag coefficient of a sphere depends on the droplet Reynolds number, $Re = \rho u_d d_c / \mu_g$. For the range of Reynolds numbers expected for drops in sprinkler sprays $C_d$ can be approximated as[8]:

$$C_d \propto \left( \rho_g u_p d_c / \mu_g \right)^{-1/2}.$$  Combining these equations with $A_d \propto d_c^2$ and $u_w \propto \dot{Q}_w / D_or^2$, $d_m/d_c$ is given by:

$$\frac{d_m}{d_c} \propto \left( \frac{g^2 \rho_w \sigma}{\rho_g \mu_g} \right)^{1/3} \frac{D_or^2}{u_p \dot{Q}_w^{2/3}}$$

(A1)
At elevated temperatures, both the gas density $\rho_g$ and viscosity $\mu_{g,r}$ are evaluated at the free stream temperature and a reference state\textsuperscript{[9]} according to the “1/3 rule”, respectively. Calculation based on air showed that the term $\rho_g \mu_{g,r}$ varied little with the present experimental conditions, and thus for convenience the density and viscosity of ambient air were used for this term when performing the regression analysis.

For the case when a single sprinkler operated inside the plume above the fire, the plume centerline velocity at the ceiling level is used for $u_p$ in Eq. (A1). For rack storage fire of the FMRC Class II commodity, the centerline velocity at the ceiling level can be calculated using the following correlation\textsuperscript{[5]}:

$$u_p = 4.25 \left( \frac{g}{\rho_c} c_{p,\infty} T_{\infty} \right)^{1/3} \left( \frac{\dot{Q}_c}{H - z_o} \right)^{1/3}$$

where $z_o = -2.4 + 0.095 \dot{Q}_c^{2/5}$,

$\dot{Q}_c$ is the convective heat release rate in kW, and $H$ is the ceiling clearance in meters.

For the four-sprinkler arrangements, $u_p$ corresponds to the maximum centerline velocity for a given fire size, which depends only on the convective heat release rate, i.e., $u_p = 2.5 \dot{Q}_c^{1/5}$.

The ratio, $\dot{M}_w / \dot{M}_p$, corresponds to the initial spray momentum flux to the plume momentum flux at the ceiling elevation. The spray momentum flux is assumed to be proportional to the water discharge momentum through the sprinkler orifice, which is equal to $\rho_w \dot{Q}_w u_w$. Assuming Gaussian profile, the momentum flux of the plume is determined:

$$\dot{M}_p = \int_0^\infty \rho_g u^2 (2\pi r) dr$$

(A-2)

$$u = u_p e^{-(r/b)^2}$$

where $r$ is the radial distance from the plume centerline and $b$ is the radius where local value of velocity is one-half of the centerline value, which is determined using the following equation\textsuperscript{[5]}:

$$b = C_b \left( \frac{c_p \rho_c}{\rho_{w,\infty} c_{p,\infty} T_{\infty}^{3/5} g^{2/5}} \right)^{1/2} \frac{T_p^{1/2} \dot{Q}_c^{2/5}}{\Delta T^{3/5}}$$

(A-3)

where $C_b$ is an empirical constant equal to 0.459. The excess gas temperature, $\Delta T$, on the plume centerline can be evaluated using the following equation\textsuperscript{[5]}:

$$\frac{\Delta T}{T_{\infty}} = \frac{11 \left( \frac{1}{g^{1/2} \rho_{w,\infty} c_{p,\infty} T_{\infty}^{3/5}} \right)^{2/3} \left( \frac{H - z_o}{\dot{Q}_c^{2/5}} \right)^{-5/3}}{10}$$

(A-4)