Influence of Stress History Function in the Schneider-concrete-model under Fire Attack

TAKESHI MORITA
Institute of Technology
Shimizu Corporation
4-17, Etchujima 3-chome, Koto-ku, Tokyo 135, Japan

ULRICH SCHNEIDER
Institute for Building Materials, Building Physics and Fire Protection
Technical University of Vienna, Karlsplatz 13/206, A-1040, Vienna, Austria

JEAN-MARC FRANSENSSEN
University of Liege
6, Quai Banning, B-4000, Liege, Belgium

ABSTRACT

A complete concrete model, named the updated Schneider-Concrete-Model which can take into account the transient creep strain and stress history, has been introduced at the 4th IAFSS Symposium in Ottawa 1994 [1]. Though the stress history is taken into account in the updated Schneider-Concrete-Model, the function for the history is simplified in [1]. This paper shows a new approach on how to take into account the stress history in a more accurate way. Sensitivity studies with the new concrete model are also done for different stress history functions by comparing the calculations with fire test results. The results of the study show that the stress level representing the stress history is not an overriding factor. The improved model named Morita-Schneider-Concrete-Model leads to a higher stiffness than the updated Schneider-Concrete-Model does, and in the case of a flexural element, the stress history function and the definition of elastic modulus do not indicate much influence on the calculation of structural fire behavior.

KEYWORDS: fire, concrete, material model, structural fire behavior, stress-strain relationship, centrally loaded element, flexural element

INTRODUCTION

A complete concrete model for high temperatures which can take into account the transient creep strain and stress history of concrete has been developed and is named the updated Schneider-Concrete-Model [1]. According to the experiments made by Schneider et al. [2,3,4], the stress history of concrete dominates the elastic modulus, the transient creep strain and the strain at maximum compressive strength. Though the stress history is taken into account in the
updated Schneider-Concrete-Model, the function for the history is simplified in [1], i.e. the stress level which determines the stress history is considered to be constant in the calculation, whereas in real structures the stress level during heating may change significantly due to thermal effects. With respect to the background described above, this paper shows:

- an improvement of the updated Schneider-Concrete-Model and
- the sensitivity and accuracy of different stress history functions in the concrete model.

The improvements of the concrete model shown in this paper concern the elastic modulus which is being harmonized to experimental elastic modulus results including load effects during heating and a new approach on how to take into account the stress history, i.e. changes in stresses due to the temperature distribution in concrete are being accounted for.

**IMPROVEMENT OF THE CONCRETE MODEL**

**Increase of Elasticity due to External Load**

Before getting into detail, the different kinds of stresses must be clearly defined in order to explain the increase of elasticity due to external load. The definition of stresses are:

- $\sigma$: actual stress in concrete as a function of time
- $\sigma_h$: constant stress due to external load prior and during heating (an equivalent stress representing the stress history which is considered to be constant in [1]).

By taking into account these definitions, a compliance function which accounts for the three strain elements, i.e. elastic strain, plastic strain and transient creep strain, according to [1,4] has the form of Eq. 1, and the definitions of elastic modulus are shown in Figure 1.

**Figure 1. Definitions of $E(\sigma, T)$ and $E(\sigma_h, T)$.**

$$J(\sigma, T) = \frac{1}{E(\sigma, T)} (1 + \kappa + \Phi)$$

$$E(\sigma_h, T) = E(\sigma_h = 0, T) \times g(\sigma_h, T)$$

where

- $T$: temperature (°C)
- $J(\sigma_h, T)$: a compliance function of stress-strain relationship
- $\kappa$: plastic strain factor
- $\Phi$: transient creep strain factor
E(\sigma_h, T) : elastic modulus

g(\sigma_h, T) : a function for the increase of elasticity due to external loads

The original g-function proposed in [4] is an empirical function and is applicable to the concrete heated under a constant stress level. The original g-function had been improved over last years [1,5]. The improved g-function has a better correlation factor between experimental and calculated results and is proposed as follows:

\[ g(\sigma_h, T) = 1 + 0.7\alpha(0.6 - \alpha) \left( \frac{T - 20}{100} \right)^2 \]  

\[ \alpha = \frac{\sigma_h}{f_c(20^\circ C)} \leq 0.3 \]  \hspace{1cm} (4)

where

\[ \alpha \] : stress level by \( \sigma_h \) during heating (constant)

\[ f_c(20^\circ C) \] : compressive strength of concrete at 20 °C

In addition to the above equations, a theoretical stress-strain relationship for concrete (Eq. 5) is used for deriving the k-factor in Eq. 1 [4]. It may be noted that Eq. 5 expresses the stress strain consisting of elastic strain and plastic strain, which is being derived by stress-strain tests at high temperatures, whereby specimens were loaded (\( \sigma_h > 0 \)) or not loaded (\( \sigma_h = 0 \)) during heating.

\[ \sigma = f_c(T) \times \frac{\varepsilon_{\text{stress}}}{\varepsilon_{\text{ult}}(\sigma_h, T)} \times \left( n - 1 + \frac{\varepsilon_{\text{stress}}}{\varepsilon_{\text{ult}}(\sigma_h, T)} \right)^n \]  \hspace{1cm} (5)

\[ \varepsilon_{\text{ult}}(\sigma_h, T) = \varepsilon_{\text{ult}}(\sigma_h = 0, T = 20^\circ C) + h(T) \times f(\sigma_h) \]  \hspace{1cm} (6)

\[ h(T) = \{ 4.2 \times 10^{-6} + (T - 20) \times 5.4 \times 10^{-9} \} \times (T - 20) \]  \hspace{1cm} (7)

\[ f(\sigma_h) = 1.0 \] for \( \left( \frac{\sigma_h}{f_c(20^\circ C)} = 0.0 \right) \]

\[ = 0.227 \] for \( \left( \frac{\sigma_h}{f_c(20^\circ C)} = 0.1 \right) \]

\[ = -0.095 \] for \( \left( \frac{\sigma_h}{f_c(20^\circ C)} \geq 0.3 \right) \]  \hspace{1cm} (8)

where

\[ \varepsilon_{\text{ult}}(\sigma_h = 0, T = 20^\circ C) = 2.2 \times 10^{-3} \]

\[ f_c(T) \] : compressive strength of concrete at \( T \) °C

\[ \varepsilon_{\text{ult}}(\sigma_h, T) \] : strain at compressive strength at \( T \) °C

\[ \varepsilon_{\text{stress}} \] : stress strain (elastic strain and plastic strain)

\[ n \] : constant ; 3 for normal concrete, 2.5 for lightweight concrete [4]

Eq. 6 is an empirical equation obtained from experiments on concrete heated under constant loads [5], like Eq. 3. Differentiating Eq. 5 with respect to \( \varepsilon_{\text{stress}} \), the elastic modulus at high temperatures can be derived according to Eq. 9. Using Eq. 9 we can get another expression accounting for the increase of elasticity due to external loads as Eq. 10.
A comparison between \( g(\sigma_h, T) \) of Eq. 3 and \( g'(\sigma_h, T) \) of Eq. 10 is shown in Figure 2. It partly indicates some differences. The \( g' \)-function of Eq. 10 should be made closer to the \( g \)-function of Eq. 3, because the \( g \)-function of Eq. 3 obtained from experiments is more precise than the \( g' \)-function of Eq. 10 which is originated from a stress - strain relationship model defined by Eq. 5. It is clear that the modulus of elasticity defined by Eq. 9 should be equal to the one defined by Eq. 2 in order to achieve similar \( g \)-functions.

\[
E(\sigma_h, T) = \frac{n \times f_c(T)}{(n-1) \times \varepsilon_{ub}(\sigma_h, T)}
\]

(9)

\[
g'(\sigma_h, T) = \frac{E(\sigma_h, T)}{E(\sigma_h = 0, T)} = \frac{\varepsilon_{ub}(\sigma_h = 0, T)}{\varepsilon_{ub}(\sigma_h, T)}
\]

(10)

FIGURE 2. ‘\( g \)’ as a function of temperature and stress level.

In Eq. 9, ‘\( n \)’ seems to be available for solving the problem. This ‘\( n \)’ is somewhat a constant for explaining the difference of material in a stress - strain relationship, for example \( n=3 \) is for normal concrete and \( n=2.5 \) for light weight concrete [4]. If the concrete at a high temperature can be assumed to be a different material compared with the concrete at room temperature, ‘\( n \)’ might also be a variable. As for Eq. 2 and Eq. 9, because these two equations must be equal to each other, ‘\( n \)’ as a function of stress history and temperature can be derived from the equality of these two equations as below:

\[
n(\sigma_h, T) = \frac{1}{1 - \frac{1}{\varepsilon_{ub}(\sigma_h, T) \times g(\sigma_h, T) \times \frac{k_c(T = 20^\circ C) \times f_c(T = 20^\circ C)}}{k_c(T) \times E(\sigma_h = 0, T = 20^\circ C)}}
\]

(11)

Using Eq. 11 the relationships Eq. 12 and Eq. 13 [1, 4] should be applied:

\[
k_c(T) = \frac{f_c(T)}{f_c(T = 20^\circ C)}
\]

\[
\begin{align*}
\text{for (}T \leq 250^\circ C) & : k_c(T) = 1 \\
\text{for (}250 \leq T \leq 750^\circ C) & : k_c(T) = 1 - 0.0018(T - 250) \\
\text{for (}750 \leq T \leq 1000^\circ C) & : k_c(T) = 0.1 - 0.0004(T - 750) \\
\text{for (}1000^\circ C < T \leq 750^\circ C) & : k_c(T) = 0
\end{align*}
\]

(12)
\[ k_c(T) = \frac{E(\sigma_h = 0, T)}{E(\sigma_h = 0, T = 20^\circ C)} \]

\[
= 1 \quad \text{for } (T = 20^\circ C) \\
= 1 - \frac{0.9}{580}(T - 20) \quad \text{for } (20 < T \leq 600^\circ C) \\
= 0.1 - \frac{0.1}{400}(T - 600) \quad \text{for } (600 < T \leq 1000^\circ C) \\
= 0 \quad \text{for } (1000^\circ C < T) 
\]

(13)

In this way we get \( n(\sigma_h, T) \) as a function of stress history and temperature. Though the values, \( E(\sigma_h = 0, T = 20^\circ C) \) and \( f_c(T = 20^\circ C) \) in Eq. (11), should be obtained from experiments for subjected concrete, \( E(\sigma_h = 0, T = 20^\circ C)/f_c(T = 20^\circ C) = 800 \) is for example applied in [6]. If \( E(\sigma_h = 0, T = 20^\circ C)/f_c(T = 20^\circ C) = 800 \) is substituted into Eq. 11, we get \( n = 2.36 \) for \( \sigma_h = 0 \) and \( T = 20^\circ C \) which is less than \( n = 3 \). Figure 3 shows \( n(\sigma_h, T) \) for \( \alpha = 0.0, 0.1 \) and 0.3 at different temperatures. It is clearly seen that 'n' shows a significant temperature dependence whereby the irregularity at 600°C may be connected with the \( \alpha \rightarrow \beta \) quartzite inversion [7] because in [2], [3] and [4] mostly concrete with quartzite aggregate was tested.

![Figure 3. ‘n’ as a function of temperature and stress level.](image)

**Proposed Solution Considering Stress History**

Though \( \sigma_h \) was considered in [1] to be constant in order to determine stress history during heating, stresses in concrete during a fire are not really constant. The proposal made here is that \( T \) and \( \sigma_h \) are considered as functions of time \( t \) in order to take into account the change of stresses in concrete during a fire. It must be mentioned that \( \sigma_h \) should be equal to \( \sigma \), because \( \sigma_h \) is dependent on time and not a constant here. \( T \) and \( \sigma_h \) are described as a function of time as follows:

\[
T = u(t) \\
\sigma_h = \sigma = v(t)
\]

(14)  
(15)
By using the two functions above, the derivations with respect to t of $\varepsilon_{ult}$ and the g-function are as follows:

$$\frac{d\varepsilon_{ult}(\sigma, T)}{dt} = \frac{\partial\varepsilon_{ult}(\sigma, T)}{\partial T} \frac{dT}{dt} + \frac{\partial\varepsilon_{ult}(\sigma, T)}{\partial \sigma} \frac{d\sigma}{dt}$$ (16)

$$\frac{dg(\sigma, T)}{dt} = \frac{\partial g(\sigma, T)}{\partial T} \frac{dT}{dt} + \frac{\partial g(\sigma, T)}{\partial \sigma} \frac{d\sigma}{dt}$$ (17)

The second terms on the right hand side in Eq. 16 and Eq. 17 should be considered carefully. A constant temperature stress history cannot allow the change of $\varepsilon_{ult}(\sigma, T)$ and g(\sigma, T), i.e. under a constant temperature and a constant load level during heating, only one stress - strain relationship can be obtained from the stress - strain test and theoretically defined with Eq. 5. The partial derivations with respect to $\sigma$ of $\varepsilon_{ult}(\sigma, T)$ and g(\sigma, T) therefore become zero. Substituting zero into the second terms on the right hand side in Eq. 16 and Eq. 17, and integrating both sides of Eq. 16 and Eq. 17 from t to $t+\Delta t$, we can get the increment of $\varepsilon_{ult}(\sigma, T)$ and g(\sigma, T) of the time step from t to $t+\Delta t$:

$$\int_t^{t+\Delta t} \frac{d\varepsilon_{ult}(\sigma, T)}{dt} dt = \int_t^{t+\Delta t} \frac{\partial\varepsilon_{ult}(\sigma, T)}{\partial T} \frac{dT}{dt} dt$$ (18)

$$\int_t^{t+\Delta t} \frac{dg(\sigma, T)}{dt} dt = \int_t^{t+\Delta t} \frac{\partial g(\sigma, T)}{\partial T} \frac{dT}{dt} dt$$ (19)

In order to apply Eq. 18 and Eq. 19 into a program using a time step integration, the following equations can be used:

If $(T_i - T_{ult,i})$ is greater than zero then:

$$\varepsilon_{ult,i} = \varepsilon_{ult,i-1} + (\varepsilon_{ult}(\sigma_{i-1}, T_i) - \varepsilon_{ult}(\sigma_{i-1}, T_{ult,i}))$$ (20)

$$g_i = g_{i-1} + (g(\sigma_{i-1}, T_i) - g(\sigma_{i-1}, T_{ult,i}))$$ (21)

If $(T_i - T_{ult,i})$ is less than or equal to zero then:

$$\varepsilon_{ult,i} = \varepsilon_{ult,i-1}$$ (22)

$$g_i = g_{i-1}$$ (23)

The scheme for solving the stress history of $\varepsilon_{ult}(\sigma, T)$ defined by Eq. 20 and Eq. 22 is shown in Figure 4 with a comparison to the scheme applied in [1]. It should be noted that $\varepsilon_{ult}$ is irreversible with regard to concrete temperature, i.e. $\varepsilon_{ult}(\sigma, T)$ for the cooling period is constant and depends on the maximum stress at the maximum concrete temperature. The g-function defined by Eq. 21 and Eq. 23 is solved by the same way as for $\varepsilon_{ult}(\sigma, T)$. Substituting Eq. 20 - Eq. 23 into Eq. 11, we get n(\sigma,T) which takes into account the change of the stress level during heating. Finally we get the stress - strain relationship which takes into account the stress history by substituting n(\sigma,T), Eq. 20 and Eq. 22 into Eq. 5, instead of substituting constant 'n' and Eq. 6 into Eq. 5.

**Example of Stress Evolution Calculated by The New Morita-Schneider-Concrete-Model**

In this paper the improved concrete model described in the foregoing sections is named Morita-Schneider-Concrete-Model. Examples applying the concrete model are shown in this section.
The conditions of the calculations are summarized as follows:
- Calculated model: plain concrete column
- Compressive strength of concrete: 50 N/mm²
- Geometry of column: (D)100 × (W)100 × (H)300 mm
- Axial load level: 15% of the ultimate strength of the column
- Thermal expansion of column: unrestrained or restrained
- Temperature history: 20°C → 500°C → 20°C (linearly heated and cooled)
- Temperature distribution in the section: homogeneous.

The evolutions of strains in the unrestrained column and the restrained column are shown in Figure 5 and Figure 6 respectively. These figures show that stress strain (elastic strain and plastic strain), thermal strain and transient creep strain can be derived separately from the concrete model. The residual stress strains after cooling down to 20°C are plastic strains, which are much less than the residual thermal strains and the transient creep strains. Comparing the strain evolution of the restrained column with the unrestrained column, the transient creep strain of the restrained column is much larger than that of the unrestrained column. The evolution of the transient creep strain leads to a significant difference concerning the residual total strain in these concrete columns. It may be noted that the proposed Morita-Schneider-Concrete-Model is able to predict the total strains with high accuracy.
FIGURE 5. Strain evolution under heating and cooling for an unrestrained column.

FIGURE 6. Strain evolution under heating and cooling for a restrained column.

APPLICATION OF THE CONCRETE MODEL

Simulation of Fire Tests - Stress history function and code

In order to compare the sensitivity and accuracy of different stress history functions of the new concrete model, three different stress-strain relationships are considered in this study. These functions are summarized in Table 1. SS-1 in Table 1 is the Morita-Schneider-Concrete-Model. SS-2 is applied in the updated Schneider-Concrete-Model described in [1]. SS-3 is the most simplified model made for this study. The functions summarized in Table 1 were implemented in a computer code “Fires-Frame I” [8, 9].

TABLE 1. Stress-strain relationships used in this study.

<table>
<thead>
<tr>
<th>σ-ε curve</th>
<th>Elastic modulus and stress level</th>
</tr>
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</table>
| SS-1      | $E : n(\sigma, T)$ of Equation (11) and Equation (9)  
|           | $\alpha : \text{Equation (20)} - (23)$ (unsteady) |
| SS-2      | $E : n(\sigma, T)$=3 and Equation (9)  
|           | $\alpha : \text{Stress level at the beginning of a concrete temperature rise (constant). For example, in the case of a centrally loaded element, } \alpha \text{ of the surface in a horizontal section may be different from } \alpha \text{ of the center, because the temperature at the center rises later than the temperature at the surface does.}$ |
| SS-3      | $E : n(\sigma, T)$=3 and Equation (9)  
|           | $\alpha : \text{Stress level just before heating (constant). For example, in case of a centrally loaded element, } \alpha \text{ in the horizontal section is homogeneous.}$ |

$E$ : modulus of elasticity, $\sigma$ : stress level
Centrally loaded element: Fire tests on centrally loaded reinforced concrete columns were carried out [1,10]. The results of the tests have already been shown and discussed in [1]. The geometry of the column is shown in Figure 7. The compressive strength of the concrete in the columns was 54.3 - 59.9 N/mm² before the fire tests. Axial load level was 15% or 30% of the ultimate strength of the concrete section of the column. Thermal expansion of the columns was unrestrained or restrained. The fire temperature - time curve prescribed in [11] was applied to the test, which is almost the same as the fire temperature - time curve prescribed in ISO834. The fire duration time of the tests was 180 minutes.

Flexural element: A fire test on a flexural reinforced concrete slab was carried out [12]. The geometry of the slab and the loading and heating condition are shown in Figure 8. The compressive strength of the concrete in the slab was 37 N/mm² before the fire test. The fire temperature - time curve prescribed in [11] was applied to the test. The fire duration time of the test was 120 minutes.
Results and Discussion

Figure 9 and Figure 10 show deformation curves of unrestrained columns during fire test. According to these figures, it can be seen that SS-2 and SS-3 lead to almost the same deformations, and that the vertical upward deformation obtained by SS-1 is larger than that of SS-2 and SS-3. Because SS-2 and SS-3 lead to almost the same deformations in spite of the different definitions for the stress level representing the stress history, it can be said that the definition of the stress level representing the stress history is not an overriding factor. The larger upward deformation obtained by SS-1 versus the deformations obtained by SS-2 and SS-3 may be due to the improvement of elastic modulus which takes into account the test results reported in [4]. The effect of elastic modulus can be foreseen from Figure 3 and Eq. 9, because ‘n’ is generally smaller than 3 if ‘n’ is calculated by Eq. 11 except around 600°C, and because smaller ‘n’ means stiffer concrete according to Eq. 9. According to Figure 9, the deformation curve during heating obtained by SS-1 is closer to the test results than that of SS-2 and SS-3. On the other hand, according to Figure 9, SS-2 and SS-3 lead to deformation curves closer to the test results than SS-1 does.

![Figure 9. Axial deformation of column (load level = 0.15, unrestrained).](image)

![Figure 10. Axial deformation of column (load level = 0.30, unrestrained).](image)

Figure 11 and Figure 12 show restraint load curves of restrained columns during fire test. According to these figures, it can be seen that SS-2 and SS-3 lead to almost the same restraint loads, and that the restraint load derived by applying SS-1 is larger than that of SS-2 and SS-3. According to Figure 11 and Figure 12, it can be said that SS-1 leads to a higher stiffness of concrete than SS-2 and SS-3 do, and that the higher stiffness of concrete defined by SS-1 is due to the improvement of elastic modulus which takes into account the test results reported in [4].
Figure 11 shows deformation curves of a flexural slab during fire test. According to this figure, SS-1, SS-2 and SS-3 lead to almost the same deformation curves, which are close to the test results. From the comparisons made here, the type of stress history function and the definitions of elastic modulus do not have much influence on the calculation of structural fire behavior in the case of flexural elements.

FIGURE 11. Restraint load on column (load level = 0.15).

FIGURE 12. Restraint load on column (load level = 0.30).

FIGURE 13. Vertical deformation at the center of the slab.
CONCLUSION

An improvement of the updated Schneider-Concrete-Model with respect to the stress history function and comparisons of the sensitivity and accuracy of three different stress history functions in the concrete model are made in this paper. The results of the study can be summarized as follows:

- Because the calculation results by using SS-2 (the updated Schneider-Concrete-Model) and SS-3 (a simplified model) do not show much difference in the deformation behavior of columns and a slab, it can be said that the stress level representing stress history is not an overriding factor.
- SS-1 (the Morita-Schneider-Concrete Model) leads to a higher stiffness than SS-2 and SS-3 do. This difference in the stiffness of concrete is due to the improvement of elastic modulus which takes into account the test results reported in [4].
- For centrally loaded elements, SS-1 leads to a good agreement with the test results in some cases, but in some cases it does not. The same tendency can be said for SS-2 and SS-3. But it is difficult to conclude from comparisons shown in this paper that SS-1 is more precise than SS-2 or SS-3. Further comparison with well documented fire tests may be necessary.
- For flexural elements, SS-1, SS-2 and SS-3 show good agreement with the test result. In the case of flexural elements, the stress history function and the definition of elastic modulus do not have much influence on the calculation of structural fire behavior.

REFERENCES

11. Enforcement No.2999 of Ministry of Construction of Japan

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