Fire Research Note
No. 807

THE FLOW OF HOT GASES ALONG AN ENCLOSED SHOPPING MALL
A TENTATIVE THEORY

by

P. L. HINKLEY

March, 1970
THE FLOW OF HOT GASES ALONG AN ENCLOSED SHOPPING MALL

A TENTATIVE THEORY

by

P. L. Hinkley

SUMMARY

The flow of smoke and hot gases in an enclosed shopping mall is discussed in general terms in another note\(^1\). The present note gives the theoretical background of that discussion which is based to a large extent on the theory of "gravity currents" investigated by Benjamin.

Formulae have been derived for calculating the rate of spread of a layer of hot gases beneath the ceiling and the depth of the layer. The depth of the spreading layer cannot exceed half the height of the mall but smoke can mix into the cold air flowing towards the fire thus effectively deepening the depth of smoke.

The formulae show encouraging agreement with the results of a large scale experiment on smoke spread carried out in Japan but more experimental work is required to establish their validity. If the theory is correct it would be necessary to install a ventilation system to ensure that occupants can escape from a mall before being overtaken by smoke in the event of fire, even if a sprinkler system is fitted in the shops bordering on the mall. Should the sprinkler system fail, smoke could spread rapidly along the mall (e.g. 100 m in 30 s).

KEY WORDS: Smoke, Spread, Shopping mall, Escape means.

_Crown copyright_

This report has not been published and should be considered as confidential advance information. No reference should be made to it in any publication without the written consent of the Director of Fire Research.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_w$</td>
<td>Window area.</td>
</tr>
<tr>
<td>$C$</td>
<td>A coefficient which would be unity for a perfect fluid.</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Coefficient of discharge beneath a roof screen.</td>
</tr>
<tr>
<td>$d$</td>
<td>Depth of layer of hot gases.</td>
</tr>
<tr>
<td>$d_c$</td>
<td>Depth of layer of hot gases beneath edge of roof screen.</td>
</tr>
<tr>
<td>$(Fr)$</td>
<td>Froude Number.</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration ($9.81 \text{ m/s}^2$)</td>
</tr>
<tr>
<td>$H$</td>
<td>Newtonian cooling constant.</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of mall.</td>
</tr>
<tr>
<td>$h_w$</td>
<td>Height of window.</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of layer of hot gases.</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass rate of flow of hot gases in layer.</td>
</tr>
<tr>
<td>$P$</td>
<td>Perimeter of fire.</td>
</tr>
<tr>
<td>$Q$</td>
<td>Rate of flow of heat in the layer of hot gases.</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>Initial value of $Q$.</td>
</tr>
<tr>
<td>$(Re)$</td>
<td>Reynolds number.</td>
</tr>
<tr>
<td>$(Ri)$</td>
<td>Richardson number.</td>
</tr>
<tr>
<td>$S$</td>
<td>Specific heat of air $(1000 \text{ J/kg}^{-1} \cdot \text{degC}^{-1})$.</td>
</tr>
<tr>
<td>$T$</td>
<td>Absolute temperature of layer of hot gases.</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Absolute ambient temperature $(293 \text{ K})$.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time.</td>
</tr>
<tr>
<td>$U$</td>
<td>((gQ/S\theta_o T W)^{1/3})</td>
</tr>
<tr>
<td>$U^1$</td>
<td>((U/T/T_o)^{1/3})</td>
</tr>
<tr>
<td>$u_o$</td>
<td>Velocity of cold air beneath layer of hot gases.</td>
</tr>
<tr>
<td>$u_s$</td>
<td>Velocity of cold air ahead of layer of hot gases.</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity of advance of layer of hot gases.</td>
</tr>
<tr>
<td>$v_m$</td>
<td>Mean velocity of gases in layer.</td>
</tr>
<tr>
<td>$v_d$</td>
<td>Velocity of advance of layer downstream when there is a forced draught.</td>
</tr>
<tr>
<td>$v_u$</td>
<td>Velocity of advance of layer upstream when there is a forced draught.</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Corrected initial velocity of advance of layer.</td>
</tr>
<tr>
<td>$W$</td>
<td>Width of mall.</td>
</tr>
<tr>
<td>$x$</td>
<td>Distance along mall measured from fire.</td>
</tr>
<tr>
<td>$x_o$</td>
<td>Value of $x$ at time $t = 0$.</td>
</tr>
<tr>
<td>$X$</td>
<td>Distance along mall in which velocity of smoke layer is halved.</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$T-T_o$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity of air $(1.6 \times 10^{-5} \text{ m}^2/\text{s} \text{ at } 20^\circ\text{C})$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of hot gases in layer.</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>Density of ambient air $(1.2 \text{ kg/m}^3 \text{ at } 20^\circ\text{C})$.</td>
</tr>
<tr>
<td>$\Delta \rho$</td>
<td>$\rho - \rho_o$</td>
</tr>
</tbody>
</table>
THE FLOW OF HOT GASES ALONG AN ENCLOSED SHOPPING MALL
A TENTATIVE THEORY

by

P. L. Hinkley

INTRODUCTION

It is envisaged that, in some town centre developments and redevelopments, pedestrian shopping precincts may include covered malls which may be a few hundred metres long and might be on the lower levels of multi-storey developments. A primary consideration in evaluating the fire hazard of any proposed scheme is the possibility of smoke and hot gases spreading rapidly along the mall and overtaking people on the escape routes. It is therefore important to be able to estimate the rate of travel of smoke along a mall.

Initially the smoke and hot gases may be confined to a stratified layer beneath the ceiling and, if the bottom is above head level, there is unlikely to be a life hazard and visibility should not be greatly reduced. However, as the leading edge of the layer of smoke and hot gases travels away from the fire it will be cooled and eventually it may become so cool that it cannot persist as a stratified layer. The mall could then become smoke-logged to floor level. Escape in those circumstances could be very difficult; this report discusses the conditions under which they might occur.

In a previous investigation of the flow of hot gases from a fire (to determine the effectiveness of roof venting) the hot gases were regarded as being confined to a relatively stagnant layer beneath the ceiling, the depth of the layer being determined by the rate of flow of gases into it from the fire and out through roof vents. This investigation provided information on the rate of "production" of hot gases by entrainment of air into a fire but was not concerned with the initial formation of the layer, which was assumed to be confined to prevent extensive horizontal spread.

The advance of a layer of hot gases in a mall is an example of "gravity currents" which have been investigated theoretically by Benjamin, who neglected the effects of viscosity and mixing between the layers. Mixing between stratified layers of different density has been studied in some detail, mainly because of its importance in the dispersal of layers of methane from beneath the roofs of coal mines and to the flow of sea water into fresh water locks. A number of experiments on the smok-logging of buildings have been carried out in Japan; one of them (in an underground car park) is described in the Appendix.
The present note tentatively combines the results of the above investigations to derive formulae for calculating the depth and rate of spread of a layer of smoke and hot gases beneath the ceiling of a mall.

Generally the hot gases in the layer will consist mainly of air heated by entrainment into the rising hot gases over the fire. Any effects on the flow due to differences in composition between the gases in the layer and air will be neglected.

For simplicity it will generally be assumed that the hot gases are flowing only in one direction along the mall. In practice the hot gases will generally be flowing in both directions away from the fire; it may be assumed that, when there is no draught, half the total flow of hot gases will be in each direction.

THE VELOCITY OF THE GASES IN THE LAYER

The velocity of the gases in the layer will be determined by the forces acting on them i.e. buoyancy, frictional and inertial forces, the transfer of momentum to the cool air by mixing and viscous forces. If the last can be neglected the flow pattern will be determined by the Froude number \( (Fr) \). If the ratio between inertial and buoyancy forces, since, to a large extent we may assume that frictional forces and the transfer of momentum by mixing are also determined by these

\[
(Fr) = \left( \frac{e - e_o}{\Delta \rho} v^2 \right)^{1/2}
\]

where:
- \( d \) is the depth of the layer
- \( v \) is its mean velocity
- \( e \) is its density
- \( \Delta \rho \) is the difference in density between the layer and the air beneath.

If \( V \) is the volume rate of flow of hot air and \( W \) is the width of the mall

\[
d = \frac{V}{Wv}
\]

\[
Fr = \left( \frac{e - e_o}{\Delta \rho} v^2 \right)^{1/2}
\]

(1)

For methane roof layers it has generally been found satisfactory to neglect the effect of density differences on the inertial term i.e. to put

\[
e - e_o = e - e_o
\]

where \( e_o \) is the density of the air beneath, so that

\[
Fr = \left( \frac{e - e_o}{\Delta \rho} v^2 \right)^{1/2}
\]

(2)

Since for methane \( e/e_o = 0.55 \) expression (2) may be applicable to air at temperatures of up to 250 deg C above ambient. Air having a temperature in the range where the approximate expression (2) is applicable will be described as "warm" air whereas air having a temperature above this will be described as "hot" air.

For "warm" air in which \( T_o \) is the absolute ambient temperature

\[
\Delta e/e = \frac{e}{T_o} \approx \frac{e}{T}
\]

(3)
Also $V \Theta = \frac{QT}{T_0} \rho_o S$ \hfill (4)

where $Q$ is the heat content of the layer crossing a plane across the mall.
$T$ is its absolute temperature
$S$ is the specific heat

From equations (2), (4) and (5)

$$Fr = \left( \frac{v^3}{S \rho_o T_0} \right)^{1/2} \frac{W}{gQ^2}$$

$$= \left( \frac{v}{U} \right)^{3/2}$$ \hfill (5)

where $U = \left( \frac{gQ}{S \rho_o T_0} \right)^{1/3}$ \hfill (6)

For hot air in which we cannot put $T \propto T_0$, we will tentatively derive an alternative expression from equations (2), (3) and (6).

$$Fr = \left( \frac{v^3}{S \rho_o T_0^2} \right)^{1/2} \frac{W}{gQ^2}$$

$$= \left( \frac{v}{U^1} \right)^{3/2}$$ \hfill (7)

where $U^1 = \left( \frac{gQ}{S \rho_o T_0^2} \right)^{1/3}$ \hfill (8)

Where the flow patterns are similar.

$\bar{v} \propto U$

or for hot air (250 deg C)

$\bar{v} \propto U^1$

This should apply to the turbulent flow of hot gases beneath the ceiling of a mall if there is no mixing with the cool air beneath and friction can be neglected.

These conditions apply to experiments $10, 11$ which have been carried out in which the gases from a fire were channelled along a corridor which was closed at one end and completely open at the other. Measurements were made of the vertical velocity profiles in the layers of fire gases and the mean velocities estimated (Table 1). A linear relationship between $\bar{v}$ and $U$ was obtained only for the gases at low temperatures but for all temperatures of gases

$$\frac{\bar{v}}{U^1} = 0.8$$ (Fig 1)
Table 1

<table>
<thead>
<tr>
<th>Distance of fire beneath ceiling</th>
<th>Width of corridor</th>
<th>Heat of layer</th>
<th>Temperature of layer above ceiling</th>
<th>U (calculated)</th>
<th>U′ (calculated)</th>
<th>From velocity measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>m</td>
<td>kW</td>
<td>deg C</td>
<td>m/s</td>
<td>m/s</td>
<td>m/s</td>
</tr>
<tr>
<td>0.66</td>
<td>1.2</td>
<td>100</td>
<td>600</td>
<td>1.3</td>
<td>1.8</td>
<td>1.2</td>
</tr>
<tr>
<td>0.66</td>
<td>1.2</td>
<td>190</td>
<td>800</td>
<td>1.6</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>2.6</td>
<td>1.83</td>
<td>200</td>
<td>800</td>
<td>1.7</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>2.6</td>
<td>1.83</td>
<td>90</td>
<td>70</td>
<td>0.7</td>
<td>0.7</td>
<td>0.5</td>
</tr>
<tr>
<td>2.6</td>
<td>3.43</td>
<td>90</td>
<td>45</td>
<td>0.9</td>
<td>0.91</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The linear relationship between \( \bar{v} \) and \( U' \) suggests that calculations and measurements of the velocities of warm air layers may be extrapolated to hot air if \( U \) is replaced by \( U' = U(T/T_0)^{\frac{3}{2}} \). At 250 deg C above ambient \( U'/U \approx 1.2 \). The value of \( \bar{v}/U' = 0.8 \) may be compared with that obtained for flow under a roof screen (i.e. a screen extending downwards from the ceiling of a building and which is intended to limit the spread of hot gases beneath the ceiling).

The volume flow rate \( V_c \) is given by

\[
V_c = \frac{3}{2} C_d W d_c^{3/2} (2g \theta / T_0)^{3/2}
\]

where \( C_d \) is the coefficient of discharge (found by experiment to be 0.6)

\( d_c \) is the depth of the layer of hot gases beneath the edge of the screen

Substituting \( d_c = V_c/\bar{v}W \)

and \( V_c \theta = QT/T_0 C_o S \)

\( \bar{v} = 0.7 (gQT/S C_o T_0^2 W)^{3/2} \)

i.e.

\( \bar{v}/U' = 0.7 \)

The advance of the 'nose' of the layer

As the 'nose' of the layer of hot gases advances along the mall it displaces cold air and the flow of the cold air must be considered when calculating the velocity of the nose. When the temperature of the layer of warm air is low the volume flow rate of warm air is approximately equal to the volume flow rate of air entrained by the rising hot gases from the fire and the air in the mall ahead of the advancing layer may be regarded as stagnant (Fig 2).

The hydrodynamical problems of the flow of the 'nose' of the layer under these conditions were investigated by Benjamin for a 'perfect' fluid neglecting viscosity and mixing. His results show that energy-conserving flow can only
occur if the rate of flow of warm air in the layer is such that \( d = \frac{h}{2} \) where 
\( h \) is the height of the mall and \( d \) is the depth of the layer (sufficiently far behind the 'nose' for the bottom of the layer to be regarded as horizontal).

He found

\[ v = \left( g \frac{\Delta \rho}{\rho_0} \frac{d}{2} \right)^{\frac{1}{2}} \]  

(9)

where \( v \) is the velocity of the nose of the layer.

For a warm air layer from equations (7) and (9)

\[ \frac{v}{U} = (2^{\frac{1}{3}})^{\frac{3}{2}} = 0.8 \]

From Benjamin's theory it follows that a rate of flow such that \( d/h > 0.5 \) could only be maintained if energy were supplied from an external source.

A fire may be considered to be a pump supplying energy to the layer but any increase in flow above the critical rate would almost certainly result in the extra volume of warm air mixing into the cold air flow.

When the rate of flow of warm air is such that \( d < \frac{h}{2} \) steady flow is possible if energy is dissipated (presumably in creating turbulence and overcoming friction at the ceiling and possibly mixing in the vicinity of the 'nose').

Benjamin showed the velocity to be given by

\[ v = \left( g \frac{\Delta \rho}{\rho_0} \frac{d}{2} \right)^{\frac{1}{2}} \left( \frac{(h-d)(2h-d)}{h(h+d)} \right)^{\frac{1}{2}} \]

(10)

He showed that both velocity and energy dissipation are a maximum when \( d/h = 0.35 \). Thus is \( 0.5 > (d/h) > 0.35 \) a slight reduction in flow rate would lead to an increase in both velocity and energy dissipation. This is an unstable situation and there may be a tendency for transition through a "hydraulic jump" to the stable condition where \( d/h < 0.35 \), the excess warm air being mixed into the cold air downstream. Thus Benjamin's work leads to the conclusion that stable layers of warm air advancing along a mall cannot occupy more than the upper half and possibly cannot occupy more than the upper third of the height of the mall.

It does however, neglect any mixing of smoke into the return flow of cold air.

**Calculation of Depth and Velocity of Advance of Layer**

For warm air equation (10) becomes

\[ \frac{v}{U} = \left( \frac{(h-d)(2h-d)}{h(h+d)} \right)^{\frac{1}{3}} \]

(11)

The depth of a spreading layer is

\[ d = \frac{V}{Wv} \]

(12)

The mass rate of "production" of hot gases by entrainment of air into the flames from a fire (\( m \)) is

\[ m = 0.05 \rho_0 (h-d)^{3/2} \left( \frac{d}{\rho_0} \right)^{\frac{1}{2}} \]

where \( \rho_0 \) is the perimeter of the fire.
If the gases are subsequently cooled so that $v \propto v_0$

$$v = 0.05P(h-d)^{3/2} \frac{g^2}{v}$$

(13)

From equations (11-13)

$$\frac{P(gh)^{3}}{UW} = 0.05 \left\{ \frac{d}{(h-d)^7/2(h+d)} \right\}$$

(14)

$P(gh)^{3}/UW$ is a dimensionless expression for the rate of flow of warm air.

For real fluids the velocity of advance of the layer of hot gases may be less than that given by equation (11) because of viscosity and frictional effects and mixing.

Equation (11) may be written

$$\frac{v}{UC} = \left\{ \frac{(h-d)(2h-d)}{h(h+d)} \right\}^{3/2}$$

(15)

where $C < 1$. Results of experiments on gravity currents reported by Benjamin appear to yield values of $C$ approaching unity in some instances. The mean may be about 0.82.

Equation (14) then becomes.

$$\frac{P(gh)^{3}}{UW} = 0.05 \left\{ \frac{d}{(h-d)^7/2(h+d)} \right\}$$

(16)

$\frac{v}{UC}$ was plotted as a function of $h/d$ (Fig 3) and $h/d$ was plotted as a function of $P(gh)^{3}/UW$ (Fig 4). From Figs 3 and 4 a relation between $v/UC$ and $P(gh)^{3}/UW$ was derived (Fig 5).

Figures 4 and 5 enable the depth and velocity of the layer of hot gases to be predicted from the dimensions of the mall and the size of the fire when $C$ is known.

The Japanese experiment with no forced ventilation is described in the Appendix. The convective heat output of the fire was probably about 300 KW (from the relative areas of the trays used by the Japanese and by Hird et al.10) giving $U = 0.83$ m/s assuming that no cooling of the layer had occurred. The initial velocity of the "nose" was about 0.4 m/s so that $v/UC \approx 0.5$.

The perimeter of the fire spreader was 8 m and the height of the car park was 3 m so that if $C = 1$

$$P(gh)^{3}/UW = 3.7$$
From Fig 5 (with $C = 0.82$ as suggested earlier) $v/U = 0.9$ which is higher than the measured value of 0.5. By trial and error it was found that the measured value of $v/U = 0.5$ was obtained if $C = 0.5$ i.e. $P(gh)^{2/3}/UW = 7.4$. Then from Fig 4 the initial layer depth $d/h = 0.24$. This may be compared with the value of $d/h$ of 0.4 measured after 2 min when the velocity of advance of the layer was slightly more than half its initial value (Fig 7).

The agreement is encouraging but further work is clearly required to test the theory; in particular the value of $C = 0.5$ is much lower than suggested by Benjamin.

**EFFECT OF A FORCED DRAUGHT**

It is assumed that there is a forced draught through the mall which is superimposed on the draught due to the air entrained by the fire. Benjamin's treatment is applicable to this situation if the velocity of the nose is measured relative to the velocity of the cold air stream ahead of the nose. If a discontinuity in depth does not occur near the fire i.e. $d/h$ is the same for flows upstream and downstream of the fire, (Upstream is taken to mean "in a direction opposite to that of the through draught"),

\[ v_u = v - u_s \quad (17) \]

and

\[ v_d = v + u_s \quad (18) \]

where $v_u$ is the velocity of the nose upstream

$v_d$ is its velocity downstream

$u_s$ is the velocity of the through draught in the mall.

In the practical situations equation 17 may not strictly apply because friction at the roof will be greater upstream than downstream. In particular when $u_s/U \rightarrow v_c/U$, $v_u \rightarrow 0$ and friction will become very small so that in effect $C$ will increase possibly to unity.

Thus the velocity of air flow required to prevent a layer of warm air backing up against the air flow in a mall may be equal to $v$ calculated from equation 15 (or Fig 5) putting $C = 1$. This gives a critical value of $u_s/U$ varying between 0.79 and 1.25 according to the layer depth which may be compared with the value of about unity found by Thomas. (12) ($u_s/U$ corresponds to the layering number defined by Bakke and Leach).

A situation equivalent to a forced draught occurs in an open ended mall when the temperature of the layer of hot air is so high that its volume flow rate cannot be regarded as equal to the volume flow rate of cold air. Then (in the absence of wind effects) there will be a flow out of the ends of the mall ($V_s$) given by

\[ V_s = V e/T_0 \]

i.e.

\[ u_s = dv e/T_0 \quad (19) \]
Conversely the effect of extracting warm air from the layer near the fire (e.g. by roof venting) will be to cause draught towards the fire

\[ u_s = V\sqrt{Wh} \]  

(20)

where \( V \) is the volume rate of venting of warm air.

MIXING BETWEEN THE HOT AND COLD LAYERS

It has so far been assumed that generally the hot layer floats on the cold layer and does not mix with it except perhaps in the vicinity of the 'nose'; molecular diffusion will of course occur but this negligible compared with turbulent mixing. Mixing between the layers is opposed by the buoyancy of the hot layer.

When the cold layer is not turbulent it may be entrained into a turbulent hot layer; the rate of entrainment can be expressed as a function of the Richardson number \((Ri)\) which may be written as

\[ (Ri) = g\theta/\sqrt{\frac{V}{\nu}} (\vec{v} - u_o)^2 \]  

(21)

It is assumed that mixing will not become important until the layer of hot gases has cooled to the extent that density differences may be neglected except for buoyancy effects (i.e. \( U^1 \) can be replaced by \( U \)).

Substituting

\[ d = V/\sqrt{\nu} \]

and

\[ V\theta = QT/T_o \rho S \]  

in equation 21

\[ (Ri) = g\theta/\sqrt{\frac{S}{\nu}} \sqrt{\frac{V}{\nu}} (\vec{v} - u_o)^2 \]  

(22)

\[ = U^3/\nu \sqrt{\frac{V}{\nu}} (\vec{v} - u_o)^2 \]  

(23)

If the cold air is stagnant,

\[ (Ri) = U^3/|\vec{v}|^2 \]  

(24)

Turbulent entrainment does not occur if \((Ri) > 0.8\) approximately; i.e. for entrainment to occur with a stagnant cold layer \(\vec{v}/U\) must be greater than 1.98.

If the velocity of the layer is governed by the velocity of the 'nose' \(\vec{v}/U\) could have a value of 1.25 for a perfect fluid (Fig 3). However with real fluids mixing is possible only if \(C > 0.96\) a value which may not be obtainable in practice. Generally the cold air layer will not be stagnant and (because its depth is large) it will usually be turbulent.

Only a small air velocity is required to induce turbulence in a typical shopping mall; if the critical Reynolds number is taken as 3,000 (by analogy with flow through a smooth duct)

\[ (Re) = \frac{u_v h}{\nu} = 3000 \]

where \( h \) is the height of the mall (assumed to be less than its width)

\( \nu \) is the kinematic viscosity of air \((1.6 \times 10^{-5} \text{ m}^2/\text{s at room temperature})\)

If the mall is 3 m high

\[ u_v = 0.016 \text{ m/s} \]  

- 8 -
When the Richardson number is less than the critical number and when both layers are turbulent they will tend to mix into each other (i.e. smoke will be mixed into the lower layer).

Bakke and Leach assumed that the mean eddy diffusivity across a layer is given by the product of the boundary layer eddy diffusivity (assuming no density gradients) and a "buoyancy coefficient" \( k \) which is a function of the Richardson number only. When \( (R_i) > 0.8 \), \( k = 0 \) (i.e. there is no mixing when \( (R_i) > 0.8 \)).

When the velocity of the cold air \( u_o \) is due only to entrainment by the flames

\[
   u_o = - \left( \frac{d}{h-d} \right) v
\]

and (assuming the velocity of the warm air layer is equal to the velocity of its nose)

\[
   (R_i) = \frac{U^3}{v^2} \left( \frac{d}{h-d} + \frac{d}{h-d} \right)^2
\]

From (15) and (26)

\[
   (R_i) = \frac{(h-d)(h+d)}{C^2 h(2h-d)}
\]

(27)

\( (R_i)C^2 \) is shown as a function of \( d/h \) in Fig. 6.

The Richardson number has minimum values when \( d/h \to 0 \) or \( d/h = 0.5 \) although the variation with \( d/h \) is only slight. If \( C \) is less than about 0.8 mixing should not occur at the horizontal boundary between a spreading layer and the cool air beneath, without external draughts. The difference between this and the mean value of 0.82 obtained by Benjamin is not significant. It is possible that \( C \) is generally low enough for mixing to be absent although further investigation is desirable.

This does not preclude the possibility of mixing occurring near the nose of the advancing layer.

If air were a 'perfect' inviscid fluid the velocity of the advancing layer of warm air relative to the velocity of the cold layer should be independent of the absolute velocity of the cold layer. To the extent that there is true in practice, a general air movement in the wall will not increase the tendency for mixing to occur. However in practice the reduction in absolute velocity of the warm air upwind of the fire will result in increased friction at the roof and hence \( C \) will effectively be increased and mixing becomes more likely.

Downwind of the fire the effect of friction at the roof will be increased. Bakke and Leach have shown that, for a given friction factor, if the layering number \( (u_o/U) \) exceeds a critical value the warm layer will be slowed to the
extent that it will be travelling more slowly than the cold air. At a still higher value of the layering number, mixing will commence and the warm layer will be dragged along by the cold air. The lowest value of layering number at which mixing can occur in a horizontal mall is \( 2 \) (ref.7).

**THE EFFECT OF COOLING**

We have

\[
U = \left( \frac{gQ}{S_0 T_0} W \right)^{\frac{1}{2}}
\]

Since \( Q \) is the rate of flow of heat in the layer, it follows that \( U \) will decrease with distance from the fire, due to loss of heat to the surroundings. Since the nose velocity is approximately inversely proportional to \( U \) the velocity of advance of the layer will decrease with distance from the fire. This means that the layer must increase in depth; it will be assumed that the rate of increase is slow enough for the bottom of the layer to be regarded as horizontal over substantial distances.

i.e. its depth is given by

\[
d = \frac{Vt}{Wl}
\]

where \( l \) is the length of the layer at time \( t \)

The heat balance equation is

\[
\frac{dQ}{dx} = -HGW
\]

where \( H \) is the Newtonian heat transfer coefficient

\( x \) is the distance measured along mall.

but \( \Theta = \frac{Q}{mS} \)

where \( m \) is mass rate of flow of hot gases

\[
\frac{dQ}{dx} = -\frac{HGW}{mS}
\]

\[
\log_e \left( \frac{Q}{Q_0} \right) = -\left( \frac{HGW}{mS} \right)(x-x_0)
\]

where \( Q = Q_0 \) at \( x = x_0 \)

but \( \frac{U}{U_0} = \left( \frac{Q}{Q_0} \right)^{\frac{1}{2}} \)

i.e.

\[
x-x_0 = \left\{ \log_e \left( \frac{U_0}{U} \right) \right\} \left( \frac{3mS}{HW} \right)
\]

The distance required to halve the velocity \( (X) \) is obtained by putting \( U_0/U = \frac{1}{2} \)

\[
X = 2.1 \text{ mS/}HW
\]

Ignoring the effect of \( d \) on \( v/U_C \) (which is shown in Fig 3)

\[
v/v_0 = U/U_0
\]

From (28)

\[
v/v_0 = \exp - \left\{ \frac{HGW(x-x_0)}{3mS} \right\}
\]

- 10 -
but \[ v = \frac{dx}{dt} \]
i.e. \[ \frac{1}{v_0} \int_{x_0}^{x} \exp\left(\frac{H}{\rho m g} (x-x_0)\right) \, dx = \int_{0}^{t} dt \]

where \[ x = x_0 \text{ when } t = 0 \]
i.e. \[ t = \frac{3m g}{H v_0} \left( \exp\left[\frac{H}{3m g} (x-x_0)\right] - 1 \right) \] (30)

From roof venting theory

\[ m \approx 0.05 \frac{P h^{3/2}}{2m} \left(1 - \frac{d}{h}\right)^{3/2} \rho g \frac{1}{2} \] (31)

Strictly \( m \) decreases as the layer becomes deeper but no allowance has been made for this. According to the above theory the rate of advance of the hot gases will eventually become very small (a few mm per second). However as the layer becomes cooler other factors such as local draughts within the mall (which may induce turbulence so causing turbulent diffusion) may become relatively more important than the buoyancy controlled flow considered in this paper.

In the Japanese experiments (appendix)

\[ (h-d) \approx 2m \]
\[ P = 8m \]
\[ m \approx 4.3 \text{ kg/s (from equation (31))} \]
\[ W = 14 \text{ m} \]
\[ H \text{ was taken to be } 20 W m^{-2} \text{ deg C}^{-1} \text{ (this makes some allowance for cooling by radiation).} \]

The distance in which the velocity would be expected to be reduced to half its original value

\[ X \approx 32 \text{ m (from equation (29))} \]

The distance-time curve for the advance of the nose of the layer of hot gases under the conditions of the Japanese experiments was calculated from equation (30) (inserting the initial velocity \( v_0 = 0.4 \text{ m/s estimated from the tangent at the origin to the experimental distance-time curve (Fig 7))}. The calculated and experimental curves are compared in Fig. 7.

Although there are considerable uncertainties in the values of \( m \), \( U_0 \) and \( H \) and only one test is involved the agreement is encouraging.
Possible patterns of flow of warm smoky gases and cold air due to a fire in a mall are illustrated diagrammatically in Fig 8. These patterns were inferred from the foregoing theory and from the results of the Japanese experiments. When there is no through draught the flow pattern will be symmetrical about a plane through the centre of the fire and only one side is shown in the diagrams.

Initially the layer will flow beneath the ceiling without mixing with the air beneath although some entrainment of smoke into the cold air near the nose of the advancing layer may occur (Fig 8a). As the velocity of the nose of the layer decreases due to cooling \( \frac{d}{h} \) will increase to maintain the rate of flow of gases. According to Benjamin \( \frac{d}{h} \) cannot exceed 0.5 and it is possible that stable flow cannot exist if \( \frac{d}{h} \) 0.35. In the Japanese experiments smoke layers deeper than this were formed. According to the theory given here it is likely that this should arise because near the nose of the warm layer smoke was entrained into the cold air where it formed a second stratified layer moving with the cold air stream and concealing the boundary of the outward flowing warm air (Fig 8b); velocity profiles would be required to test the theory. The depth of this reverse flow of smoke increased as the velocity of the nose decreased, implying increased mixing at the nose. After the nose had travelled 70 m in the Japanese experiments smoke logging extended to ground level near the fire and the zone moved along the mall against the cold air flow (Fig 8c). This may have been due to warm air being supplied to the layer at a greater rate than it could flow along the layer so that local circulation occurred near the fire. This might be expected to occur after \( \frac{d}{h} = 0.5 \).

The initial value of \( \frac{P(gh)^2}{UW} \) in the Japanese experiments has been estimated to be 7.2. This will have increased to 22, corresponding to \( \frac{d}{h} = 0.5 \) (Fig 4), when \( U \) has fallen by a factor of 3.5, which from equation (29) occurs after the nose has travelled 60 m. This is close to the 70 m travel before smoke logging reached the ground in the Japanese experiments.

**EFFECT OF OPEN-FRONTED SHOPS LINING THE MALL**

When the tops of the openings are at the level of the ceiling in the mall the smoke will spread through the shopping centre in much the same way as in a large open compartment. In many instances the fascias over the shop fronts will form a channel along which the hot gases will flow until the depth of the layer exceeds the depth of the fascias. The open shop fronts may provide alternative paths for the flow of cold air thus reducing its velocity. This may be likened to the effect of increasing the height of the mall and hence \( v/U \) may be increased.

**EFFECT OF THE TEMPERATURE OF THE LAYER OF HOT GASES**

The foregoing theory assumes that density differences between the gases in the layer and the cold air beneath can be neglected except for their effect on
buoyancy i.e. the volume of the gases in the layer is not significantly different from the volume of air entrained by the fire. This approximation is more valid as the air in the layer becomes cooler. As explained earlier this will increase U, which may be replaced by \( U' = U(T/T_0)^3 \), and with an open-ended mall will also result in the velocity of advance of the hot air layer being further increased by a factor approaching \( d \theta/T_0 \) (from equation 19).

In addition the flow rate \( V \) will be increased by a factor \( T/T_0 \) so that the layer will be deeper than would be calculated from the formulae given in this paper. In practice it may be possible to make some correction for these errors by adjusting the value of \( C \) but their practical importance requires further investigation.

**EFFECT OF LOCALLY HIGH AIR VELOCITIES**

Air velocities may be high locally at some points in the mall, this may be due to forced ventilation air inlets or extracts or to wind blowing through doors or windows. Gusts or eddies of wind may cause high velocities varying both with time and position within the mall. Thus the layering number may be locally high and at those positions mixing of smoke laden hot gases into the cool air layer is possible; this smoke will subsequently drift with the cool air layer.

**EXAMPLES**

Although the theory may require modification in the light of experimental evidence it is instructive to use it to obtain tentative estimates of the travel of smoke along a mall for two different sizes of fire.

No allowance will be made for the time taken for the fire to grow to its maximum size or of any delay due to the smoke spreading within the shop before it reaches the mall.

The mall is assumed to be 4 m high and 6 m wide with flow in both directions so that its effective width \( W \) is 12 m. Reference will be made to flow in one direction only but exactly the same flow will occur in the opposite direction.

1. A fire 2 m x 2 m in the base area with flames 2 m high occurs in a shop with an open front. This may be of the order of the area to which a sprinkler system would confine a fire.

The convective heat output of such a fire would be about 1 MW (1000KW).

If

\[
C = 0.5
\]

\[
U = 1.32 \text{ m/s}
\]

and

\[
P(gh)^{1/2}/UWC = 6.35
\]

From Figs 4 and 5 the initial layer velocity is 0.7 m/s and the initial depth of the layer is 1 m.
If the heat loss coefficient is 20 W m\(^{-2}\) deg C\(^{-1}\) it may be calculated from equation 28 that the "nose" will have reached 50 m in \(1\frac{1}{2}\) min, 70 m in \(2\frac{1}{2}\) min and 100 m in 4 min. The calculated distance of travel after \(\frac{1}{2}\) hour is 250 m i.e. the flow due to buoyancy is very small and other factors such as local draughts will be controlling the flow.

When the layer has travelled about 30 m (in about 1 min) the depth of the outflowing layer will have increased to about 1.3 m and it is likely that, as the layer travels on, considerable smoke will recirculate in the top of the return flow of cool air.

(2) A fully developed fire occurs in a shop with an open front 6 m \(\times\) 3 m high.

The formulae must be modified for this situation since the hot gases may be at a temperature of at least 900 deg C above ambient. The mass rate of flow of hot gases is roughly given by

\[ m = 0.5 \cdot \frac{A \cdot k \cdot h}{w^2} \textrm{kg/s} \]

where \(A\) is area of window (m\(^2\))
\(k\) is its height (m)

i.e. \(m \approx 16\) kg/s

The convective heat output is about 2 kJ per g of air

i.e. \(Q \approx 30MW\)

\[ U^1 = 6.49\textrm{ m/s} \]

Taking \(v = U^1\) C

and \(C = 0.5\)

\(v = 3.2\) m/s

To this must be added the general velocity in the mall.

\[ u_s + v = 8\textrm{ m/s} = v_i \]

where \(v_i\) is the corrected initial velocity of advance of the layer.

The layer depth is

\[ d = \frac{V}{v_i} \]

\[ = \frac{m \cdot q \cdot T}{0 \cdot v_i} \]

\[ = 0.8\textrm{ m} \]

i.e. \(H/d = 0.2\)

Taking a high cooling coefficient (60W/m\(^2\)) because of the elevated temperature; from equation (30) the layer will advance roughly 100 m in 30 s and 200 m in \(2\frac{1}{2}\) m. After roughly 1.3 s the layer depth will have increased to 2 m and smoke logging to ground level may start near the fire.
Assuming the theory to be correct, the first example shows that, even if the size of the fire is restricted by sprinklers, exits from the mall within 70 m of the fire in both directions could be difficult to use because of smoke within 2½ min. This implies that escape from the mall could be hindered by smoke and that a natural or forced draught ventilation system should be installed, even if the shops are protected by sprinklers.

The second example shows that, again assuming the theory to be correct, a dangerous rate of spread of smoke could occur if the sprinkler system failed in a relatively small compartment.

CONCLUSIONS

The results of a theoretical investigation by Benjamin into the flow of "gravity currents" in inviscid fluids have been applied to the flow of hot smoky gases from a fire in a covered shopping mall.

Formulae have been derived for calculating the rate of spread of the layer of hot smoke beneath the ceiling and its depth.

The rate of advance of the layer depends only slightly on the rate of entrainment of air by the fire (the perimeter of the fire) but is roughly proportional to the cube root of its heat output and is roughly inversely proportional to the cube root of the width of the mall.

The depth of the layer in a mall of given height is roughly proportional to the perimeter of the fire and inversely proportional to the product of the velocity of the layer and the width of the mall.

When the fire has grown beyond a critical size, for which the layer of hot gases occupies half the height of the mall, the flow becomes unstable near the fire and smoke logging to ground level may occur.

Even when the fire is smaller than this critical size there may be some mixing of hot smoky gases from the nose of the advancing layer into the flow of cold air towards the fire thus deepening the layer of smoke.

The formulae show encouraging agreement with the results of an experiment carried out in Japan on the smoke logging of an underground car park but more experimental work is required to test the validity of this application of Benjamin's results and to verify that the coefficient C is only slightly less than unity.

- 15 -
A number of tests on the spread of smoke in buildings both with and without forced ventilation have been carried out in Japan. Unfortunately the work is written in Japanese so that until a translation becomes available any conclusions drawn by them cannot be taken to account in this paper and the following notes are based on the Japanese results as given in the figures to the reports.

Three tests were carried out in part of an underground car park 3 m high; this was divided up to form a series of chambers connected by large openings (Fig. 9). The mean width of the chambers was about 14 m and the total length was about 150 m. In test 1 a forced extract system was in operation with air admitted in the position shown in Fig. 9, in test 2 there was a combined inlet-extract system in operation while in test 3 there was no forced ventilation. The fire was produced by burning alcohol in a 1 m square tray 1 m above which was a 2 m square "spreader", a chemical "smoke" was introduced into the hot gas stream. This fire burnt for about 10 min i.e. it probably represents the situation where a fire burns up but sprinklers open and it is extinguished in 10 min.

Diagrams illustrating the spread of smoke during the tests are given in the reports (that for test 3 is given in Fig. 10) the spread of smoke at ceiling level (taken from those diagrams) is shown in Fig. 11. Initially the smoke spread out as a layer of the order of 1 m deep beneath the ceiling - the rate of travel of the leading edge being about 0.4 m/s. As the layer became longer its rate of travel decreased and it apparently increased in depth until the compartment became smoke-logged from ceiling to floor near the fire. Measurements of temperature showed that a stratified layer of hot gases 1-2 m deep persisted near the burner until some time after the fuel had burnt out (Fig. 12).

The smoke distribution at the same point (Fig. 13) did not have the same distribution as that of temperature. When there was no forced ventilation (test 3) there appeared to be a relatively stagnant layer extending to at least 50 m for the fire between the hot and cold layers (at about head height). This became smoke logged after about 6 minutes and persisted for at least ½ hour after the fuel had burnt out. This layer was not so marked in test 2 and was rapidly dissipated when the fuel had burnt out by the forced ventilation. In test 1 the stagnant smoke layer did not form.
REFERENCES


FIG. 1. MEAN VELOCITY AS A FUNCTION OF $u'$. 

$u' = \left( \frac{gQT}{\rho_e T_0^2 W} \right)^{\frac{1}{3}} \text{ m/s}$
FIG. 2 IDEALIZED FLOW PATTERN
FIG. 3 VELOCITY AND LAYER DEPTH
FIG. 4 LAYER DEPTH AND FLOW RATE
FIG. 5 VELOCITY AND FLOW RATE

Dimensionless flow rate at which \( d/h = 0.5 \)
FIG. 6 RICHARDSON NUMBER AND LAYER DEPTH
FIG. 7 DISTANCE OF SPREAD: COMPARISON WITH THEORY
FIG. 8 PROBABLE FLOW PATTERNS IN A MALL WITH A WARM AIR LAYER ADVANCING ALONG IT
FIG. 9 PLAN OF UNDERGROUND CAR PARK USED IN JAPANESE TESTS
FIG. 10 SPREAD OF SMOKE IN JAPANESE UNDERGROUND CAR PARK (TEST 3)
Approximate duration of fire

Passage direction reversed (see fig. 9)

FIG. 11 TRAVEL OF UPPER EDGE OF SMOKE LAYER
FIG. 12 VERTICAL TEMPERATURE DISTRIBUTION
AT ABOUT 12m FROM FIRE (TEST 3)
FIG. 13 VERTICAL SMOKE DENSITY DISTRIBUTION AT ABOUT 12m FROM FIRE