STRESS-STRAIN CURVES OF CONCRETE AT HIGH TEMPERATURE - A REVIEW

by

R. BALDWIN and M. A. NORTH

October 1969
STRESS-STRAIN CURVES OF CONCRETE AT HIGH TEMPERATURE - A REVIEW

by

R. Baldwin and M. A. North

SUMMARY

This note reviews some data on the effect of temperatures (up to 700°C) on the relationship between stress and strain for concrete under compression. It is shown that if all the data are normalised to the stress and strain at the peaks of the various stress-strain curves for different temperatures (the compressive strength) the curves are virtually the same.

KEY WORDS: Concrete, Stress-strain curve, Temperature.
STRESS-STRAIN CURVES OF CONCRETE
AT HIGH TEMPERATURE - A REVIEW

by

R. Baldwin and M. A. North

INTRODUCTION

This paper reviews some of the data in the literature relating to the effect of temperature on the elastic properties of concrete under compression at high temperatures. The only complete stress-strain curves appear to be those of Furamura, and these are the main concern of this paper. There are, however, other data: Malhotra has measured the effect of temperature on the compressive strength of concrete, whilst Saemann and Washa, Philleo, and Cruz have investigated the effect of temperature on the modulus of elasticity. These data will be used for the purposes of comparison with those of Furamura where possible.

Malhotra, Philleo and Cruz have all shown that the physical properties of concrete are strongly influenced by various factors such as the aggregate, age etc. In view of this, the numerical value of the elastic constants derived from Furamura's data apply only to the particular specimens tested, and hence the emphasis in this paper will be on qualitative results, and in particular the way in which the stress $\sigma$ and strain $\varepsilon$ varies with temperature.

FURAMURA'S DATA

Some typical stress-strain curves of Furamura are shown in Fig. 1 for temperatures ranging from 300°C to 700°C. These experiments were conducted with cylindrical specimens placed under compression in a furnace, the strains being derived from measurements of the change of length of the specimen. The loads were cycled during the course of each measurement, and although on relaxing stressed specimens the strain did not return to zero, upon restressing the original stress-strain curve was again resumed at the point at which relaxation took place. This implies that the concrete acquires a permanent set during stressing, and that the stress-strain curves in Fig. 1 are not reversible; that is, they apply only in the direction of increasing strain.
NORMALISATION OF DATA

The curves of Fig. 1 exhibit a rather complicated stress-strain temperature relationship, non-linear over most of the range. However, some important conclusions may be drawn by imposing the following transformation on the data. For each temperature let $\sigma_m$ be the stress at the peak of the stress-strain curve and let the strain at this point be $\varepsilon_m$. We now plot $\sigma/\sigma_m$ against $\varepsilon/\varepsilon_m$ for each temperature and the resulting plot is shown in Fig. 2. It is clear that the effect of this transformation is to bring all the curves for different temperatures together so that they lie on the same curve.

This result has great significance because it implies that the stress-strain curves for high temperatures can be derived entirely from the stress-strain curves measured at room temperatures together with the variation with temperature of the compressive strength of the material, corresponding to the peak of the curve, and the strain at this point.

COMPRESSIVE STRENGTH AT DIFFERENT TEMPERATURES

The compressive strength of the specimens at different temperatures is plotted in Fig. 3, derived by finding the maximum stress of each stress-strain curve. This shows that the compressive strength remains nearly constant until around 300-400°C when there is a dramatic decrease in strength. Malhotra has investigated the compressive strength of concrete at high temperatures, and a typical curve from his paper is plotted in Fig. 3 for the purposes of comparison. It should be noted that this curve may undergo considerable modification both in location and slope for varying aggregates, age etc. of the tested material.

The strain $\varepsilon_m$ at the peak of the stress-strain curves is plotted for different temperatures in Fig. 4. This is the only data of its kind available in the literature so no comparisons are possible.
MODULUS OF ELASTICITY, $E$

It has been shown in Fig. 2 that
\[ \frac{\sigma}{\sigma_m} = f\left(\frac{\varepsilon}{\varepsilon_m}\right) \]
to a very good approximation, and certainly within the experimental accuracy of the data. The stress-strain curve is distinctly non-linear, and in the range $0 < \varepsilon < \varepsilon_m$ the following is an adequate representation of the data
\[ \frac{\sigma}{\sigma_m} = \frac{\varepsilon}{\varepsilon_m} \left(2 - \frac{\varepsilon}{\varepsilon_m}\right) \]

An alternative representation of the data over the whole range of $\varepsilon/\varepsilon_m > 0$ due to Thomas is as follows:
\[ \frac{\sigma}{\sigma_m} = \frac{\varepsilon}{\varepsilon_m} \left(1 - \frac{\varepsilon}{\varepsilon_m}\right) \]

Over the early part of the curve, for small values of $\varepsilon/\varepsilon_m$, the curve is approximately linear and then
\[ \frac{\sigma}{\sigma_m} = K \frac{\varepsilon}{\varepsilon_m} \]
so that
\[ \sigma = \left(K \frac{\sigma_m}{\varepsilon_m}\right) \varepsilon \]

Writing $E$ for the modulus of elasticity, where $\sigma = E \varepsilon$
we now have
\[ E = K \frac{\sigma_m}{\varepsilon_m} \]

The constant $K$ in this equation can be determined from the values of $E$ and $\sigma/\sigma_m$ at room temperature, with $E$ derived from the slope of the stress-strain curve for small values of $\varepsilon$.

Then $K = 2.18$ and $E = 2.18 \frac{\sigma_m}{\varepsilon_m}$

Using this equation and deriving $\frac{\sigma_m}{\varepsilon_m}$ and $\varepsilon_m$ from the stress-strain curves, the variation of $E$ with temperature can be determined. This is plotted in Fig. 5. Also plotted on this curve are the corresponding values of $E$ derived by measuring the initial slope of the stress-strain curves, where for the sake of uniformity the part of the curve lying between zero and one quarter compressive strength has been taken to be linear. This is a reasonable approximation for all temperatures. The agreement between the two sets of points is remarkable.
Measurements of 'E', the modulus of elasticity for concrete at different temperatures, have been reported by Cruz and the average of his curves is plotted in Fig. 6, together with the data of Furamura. Cruz remarks that an exponential function will fit the plot of E against T. However, the evidence of this in Furamura's data is slight and a straight line is adequate over most of the range, particularly for T > 100°C, implying that E decreases linearly with temperature. The difference in slope in the region T < 100°C is presumably due to the presence of moisture. The data of Furamura lie well within the limits derived by Cruz for concrete of different materials.

CONCLUSIONS

1. The stress-strain curves for concrete under compression at high temperatures measured by Furamura show that the curves are of the form

\[ \frac{\sigma}{\sigma_m} = f\left(\frac{\varepsilon}{\varepsilon_m}\right) \]

where \( f \) is a functional form independent of temperature, and \( \sigma_m, \varepsilon_m \) are the strain and stress at the peak of the stress-strain curve for a given temperature, and are themselves therefore function of temperature. In particular, for the range \( 0 < \varepsilon < \varepsilon_m \)

\[ \frac{\sigma}{\sigma_m} = \frac{\varepsilon}{\varepsilon_m} \left(2 - \frac{\varepsilon}{\varepsilon_m}\right) \]

or alternatively, for \( \varepsilon > 0 \)

\[ \frac{\sigma}{\sigma_m} = \left(\frac{\varepsilon}{\varepsilon_m}\right)^{1-\frac{\varepsilon}{\varepsilon_m}} \]

2. This result implies that the stress-strain curve for concrete at high temperatures can be derived from the stress-strain curve at room temperature together with the location of the maximum of the stress-strain curve at high temperatures - the compressive strength of the material.

3. The modulus of elasticity at temperature T can be derived from the expression

\[ E = K \frac{\sigma_m}{\varepsilon_m} \]

where K is a constant which can be derived from the stress-strain curve at room temperature. For Furamura's data K = 2.18. This method is in good agreement with values of E derived from measuring the slope of stress-strain curves at different temperatures.

4. The modulus of elasticity appears to decrease approximately linearly with temperature, particularly when T > 100°C.

5. The data of Furamura is in agreement with the measurements of other authors.
REFERENCES


4. HILLEG, R. Some physical properties of concrete at high temperatures. Journal of the American Concrete Institute, April 1958.


6. THOMAS, P. H. Personal communication.
\[ \frac{\sigma}{\sigma_{\text{max}}} = \frac{\varepsilon}{\varepsilon_{\text{max}}} (2 - \frac{\varepsilon}{\varepsilon_{\text{max}}}) \]

**FIG. 2. NORMALISED STRESS STRAIN CURVES**

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°C</td>
<td>X</td>
</tr>
<tr>
<td>70°C</td>
<td>+</td>
</tr>
<tr>
<td>100°C</td>
<td>o</td>
</tr>
<tr>
<td>300°C</td>
<td>△</td>
</tr>
<tr>
<td>400°C</td>
<td>●</td>
</tr>
<tr>
<td>500°C</td>
<td>▲</td>
</tr>
<tr>
<td>600°C</td>
<td>■</td>
</tr>
<tr>
<td>700°C</td>
<td>□</td>
</tr>
</tbody>
</table>

Data taken from the stress strain curves of Furamura
FIG. 3. COMPRESSIVE STRENGTH AS PERCENTAGE OF NORMAL STRENGTH
FIG. 4. VARIATION OF $\varepsilon_{\text{max}}$ WITH TEMPERATURE

Data taken from Furamura's stress strain curves
FIG. 5. COMPARISON BETWEEN TWO ESTIMATES OF MODULUS OF ELASTICITY
FIG. 6. MODULUS OF ELASTICITY