IGNITION BEHAVIOUR OF HEATED WOOD SURFACES

by

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SUMMARY

This report which will be mainly of interest in theoretical calculations on ignition and the spread of fire deals with two aspects of the ignition of wood. In the first a conventional approximation is used in calculating the ignition hazard. It is shown that an effective constant heat transfer coefficient can be defined to accommodate the theoretical temperature dependent coefficient. Computer calculations confirm that the approximation is satisfactory for the range of conditions of most practical interest.

The second aspect concerns the rate at which volatiles are emitted at ignition. This is some guide to the phase when, after ignition, the burning wood itself acts as a fuel assisting further fire spread. Recently published data of Koohyar are analysed. This analysis shows that, although perhaps less precise than defining ignition by the attainment of a critical surface temperature, a critical rate of emission of volatiles of the order of 6 g m\(^{-2}\) s\(^{-1}\) is a suitable alternative criterion.

KEY WORDS: Pyrolysis, Ignition, Temperature, Wood, Heat transfer, Correlation.

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NOMENCLATURE

\[ E \] : pseudo-activation energy of the rate of volatile emission, calculated from the surface temperature

\[ f \] : pre-exponential frequency factor

\[ F \] : dimensionless pre-exponential frequency factor

\[ H \] : convective heat transfer coefficient

\[ H' \] : dimensionless convective heat transfer coefficient

\[ I \] : energy flux incident at surface (radiation + convection)

\[ k \] : thermal diffusivity

\[ K \] : thermal conductivity

\[ \dot{m}'\prime \] : rate of volatile emission per unit volume

\[ \dot{m}'\prime\prime \] : rate of volatile emission per unit area

\[ M \] : dimensionless rate of volatile emission per unit area

\[ P \] : dimensionless temperature

\[ q \] : heat flux density

\[ R \] : gas constant

\[ t \] : time

\[ T \] : dimensionless time

\[ v \] : velocity of char base

\[ w_0 \] : initial proportion of volatiles

\[ x \] : distance from the heated surface

\[ X \] : dimensionless distance from the heated surface
\( \varepsilon \) : emissivity

\( \mu \) : permeability along the grain

\( \Theta \) : absolute temperature

\( \rho \) : density

\( \sigma \) : Stefan's constant

Note:

The dimensionless quantities \( H', M, P, T, X \) are not formulated in the same way in all the problems discussed.

Subscripts

- \( o \) : initial value;
- \( \infty \) : value after long period;
- \( s \) : value at either surface;
- \( 1 \) : value at the heated surface;
- \( 2 \) : value at the unheated surface;
- \( \text{ch} \) : value at the charring zone
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1. INTRODUCTION

Wood is the most widely used combustible material. Its behaviour in fires is therefore of much interest. In particular, it is important to be able to estimate the contribution that wood is likely to make to the spread of a fire. Heated wood surfaces can contribute to the growth of fires by providing combustibles, by radiating heat to other surfaces, and by allowing flames to spread along the surface. The present study considers the factors affecting the temperature of a heated wood surface, the rate of evolution of volatiles from such a surface, and the conditions for ignition. Although in the past much work, experimental and theoretical, has been done on the first of these - the temperature - the second, which is very relevant to the contribution of wood to the fuel supply, has received much less attention.

The analysis of the problem has been simplified by neglecting the variation of the thermal properties of the wood with temperature or time, self heating of the wood, diathermacy, the variation of emissivity with wave length, and the convective heat transfer within the wood.

2. SURFACE TEMPERATURE

Relationships have been derived in the past for the variation with time of the surface temperature of a solid receiving heat by convection or radiation. These treatments have usually allowed for radiation cooling by making a mean equivalent addition to the convective heat transfer coefficient. Such methods can give the transient rise approximately, particularly when radiation from the heated surface is not very important. However, when it is important, in particular near threshold conditions and when it is desired to know the amount of this radiation, the non-linearity with temperature of radiation emission should preferably be taken into account in calculating the surface temperature. Radiation from heated surfaces is important in the spread of fire in compartments. Since such radiation is proportional to the fourth power of the temperature of the radiating surface, any estimation of the intensity of radiation requires the surface temperature to be known accurately. For these reasons means of calculating the surface temperature of a heated solid cooling by radiation and convection were considered and some values calculated.
It was assumed that all the absorption of heat occurs at the surface (i.e. that there is no diathermacy) and that the emissivity of the surface is constant. These conditions are satisfied by a blackened surface and are usually applicable once the wood has charred or darkened.

In view of the complex nature of the equations and their non-linearity it is not possible to solve them using Fourier transforms or generally to obtain analytical solutions. For this reason the results are given in a form which can conveniently be solved numerically by a computer. This method has the additional advantage that it can easily be adapted to cases of varying incident heat flux.

2.1. Semi-infinite solid

The surface temperature, $\theta_s$, of a semi-infinite solid into which there is a constant heat flow of flux density $q$ is given by

$$\theta_s = \theta_o + q \left( \frac{4kt}{\pi} \right)^{1/2}$$

where $K$ is the thermal conductivity, $k$ the thermal diffusivity, $\theta_o$ the initial temperature, and $t$ the time.

If $q$ becomes $q + \delta q$ after time $t'$ then, by the principle of superposition

$$\theta_s = \theta_o + q \left( \frac{4kt}{\pi} \right)^{1/2} + \frac{\delta q}{K} \left( \frac{4k(t-t')}{\pi} \right)^{1/2}$$

Thus, if $q$ varies,

$$\theta_s = \theta_o + \frac{q_o}{K} \left( \frac{4kt}{\pi} \right)^{1/2} + \int_{t'}^t \frac{1}{K} \left( \frac{dq}{dt} \right) t-t' \left( \frac{4k(t-t')}{\pi} \right)^{1/2} dt'$$

where $q_o$ is the initial heat flux density.

Alternatively, the surface temperature after time $t$ due to heat flux density $q$ lasting for a time $dt'$ is given by

$$\theta_s = \theta_o + \frac{q}{K} \left( \frac{k}{\pi t} \right)^{1/2} dt'$$

giving, for varying $q$,

$$\theta_s = \theta_o + \int_{t'}^t \frac{q}{K} \left( \frac{k}{\pi(t-t')} \right)^{1/2} dt'$$

Consider a surface receiving heat at a net rate given by

$$q = \epsilon I - H(\theta_s - \theta_o) - \epsilon \sigma (\theta_s^4 - \theta_o^4)$$

-2-
where $H$ is the convective heat transfer coefficient

$\varepsilon$ is the emissivity

$\sigma$ is Stefan's constant

Substituting for $q$ from equation (5) into equation (4) gives

$$P_s = P_o + \int_0^{T} (1 - H' (P_s - P_o) - (P_s^0 - P_o^0)) (T - T')^{1/2} dT'$$ 

(6)

where

$$P = 8 (\sigma / I)^{1/4}$$

(7a)

$$T = \frac{4kt}{r} \varepsilon^2 \frac{1}{K} \left( \frac{\sigma}{r} \right)^{1/2}$$

(7b)

$$H' = \frac{H}{\varepsilon I} \left( \frac{\sigma}{r} \right)^{-1/2}$$

(7c)

The integral equation (6) can be solved numerically. Thus $P_s$ can be expressed as an equation involving four dimensionless variables $P_s$, $P_o$, $T$ and $H'$.

If $P_o$ and $H$ are known then $P_s$ is a function of $H'$ given by equation (8), and $P_s$ can be expressed as a function of $T$ and $H'$ only, which can be plotted as a family of curves for constant values of $H'$.

Values of $P_s$ versus $T$ were computed for three values of $H'$, and are shown in Figs (1) and (2) for $\varepsilon (\sigma / H)^{1/3} = 0.35$. Values for $\varepsilon (\sigma / H)^{1/3}$ equal to 0.45 were also calculated but are not shown.

It can be shown that with cooling only by convection the surface temperature is given by

$$\theta_s = \theta_o + (\varepsilon I / H)(1 - \exp \frac{\beta^2}{1 - \exp \varepsilon \sigma \theta \beta}$$

(9)

where $\beta = (H / K) (kt)^{1/2}$

$\exp \frac{\beta^2}{1 - \exp \varepsilon \sigma \theta \beta}$ is very nearly equal to $\pi^{-1/2} \beta^{-1}$ for $\beta$ greater than 5 and approximately so for $\beta$ greater than 1. This approximation gives

$$\theta_s = \theta_o + \varepsilon \frac{I}{H} \left( 1 - \frac{K}{H} \left( \frac{1}{\gamma r} \right) (kt)^{-3/2} \right)$$

(10)

From equation (10)

$$P_s = P_o + H^{1/2} - \gamma r^{-1} H^{1/2} - 2 \gamma^{-3/2}$$

(11)

where $P, T, H'$ are the dimensionless quantities given by equations (7a) to (7c).
From equation (11):
\[ H' = (P_\infty - P_o)^{-1} \]

or
\[ H' = \left( -\frac{\pi}{\alpha} \frac{P_o}{a(T^{-\frac{1}{2}})} \right)^{-\frac{1}{2}} \]  

(12)

(13)

where \( P_o = P_1 \) at \( T = 1 \).

Equations (12) and (13) can be used to estimate effective values of \( H' \) to be taken when radiation transfer is treated as linear with temperature.

Values for \( \xi_o \left( \varepsilon' \sigma/H \right)^{\frac{1}{2}} \) equal to 0.35 and 0.45, derived from values of \( P_o \) calculated by the use of equation (6), are given below:

<table>
<thead>
<tr>
<th>( \xi_o ) ((\varepsilon' \sigma/H)^{\frac{1}{2}} )</th>
<th>( H' ) Equation 7c</th>
<th>( P_o ) Equation 8</th>
<th>( P_\infty P_1 )</th>
<th>Effective value of ( H' ) From(12) From(13) Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>0.10</td>
<td>0.163</td>
<td>0.979</td>
<td>0.803</td>
</tr>
<tr>
<td></td>
<td>0.37</td>
<td>0.251</td>
<td>0.931</td>
<td>0.785</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>0.283</td>
<td>0.906</td>
<td>0.774</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
<td>0.392</td>
<td>0.812</td>
<td>0.732</td>
</tr>
<tr>
<td>0.45</td>
<td>0.10</td>
<td>0.209</td>
<td>0.981</td>
<td>0.819</td>
</tr>
<tr>
<td></td>
<td>0.37</td>
<td>0.323</td>
<td>0.940</td>
<td>0.815</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>0.366</td>
<td>0.921</td>
<td>0.811</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
<td>0.503</td>
<td>0.867</td>
<td>0.804</td>
</tr>
</tbody>
</table>

These effective values of \( H' \) are the values which should be taken in equation (11) if all heat loss is treated as linear. Equation (11) is linear between \( P_o \) and \( T^{-\frac{1}{2}} \) and the computed results in Fig. (2) are nearly so, which justifies the use of an effective \( H' \). Figure (2) shows that the above values of \( H' \) can be used for \( T \) greater than 0.5 or \( P \) greater than 0.7.

If \( H = 13 \, \text{W m}^{-2} \, \text{K}^{-1} \), \( I = 16 \, \text{kW m}^{-2} \), \( \xi_o = 300 \, \text{K} \), for example, then
\[ H' = 0.37, \xi_o \left( \varepsilon' \sigma/H \right)^{\frac{1}{2}} = 0.45. \]

The effective value of \( H' \) is 1.61 and the effective value of \( H \) is 34 \, \text{W m}^{-2} \, \text{K}^{-1}. \] This value is in good agreement with values obtained by Koohydr.

The approximately linear variation of \( \xi_o \) with \( T^{-\frac{1}{2}} \) can be used to estimate the equilibrium surface temperature from experimental results. For this purpose the surface temperature should be plotted against \( T^{-\frac{1}{2}} \) and the temperature at \( t = \infty \) found by extrapolation.
2.2. Heated slab

The analytical solution for a heated slab is complicated, even when non-linear cooling terms are ignored. Accordingly a numerical method of computation was worked out, and is given in the Appendix.

3. EVOLUTION OF VOLATILES

It is important to know the rate of evolution of volatiles from heated wood surfaces since this quantity can determine the size of the fire and the rate of flame spread.

To simplify the analysis it was assumed that there is only one decomposition reaction. Tinney assumes that the evolution of volatiles per unit volume, \( \dot{m}'\), follows a first order Arrhenius decomposition equation given by

\[
\dot{m}' = (\rho - \rho_\infty) f \exp \left(-\frac{E}{R\Theta}\right)
\]

(14)

where \( \rho \) is the density and \( \rho_\infty \) the density after all the volatiles are driven off.

From equation (14),

\[
\frac{d}{dt} (\rho - \rho_\infty) = \dot{m}'
\]

\[
= (\rho - \rho_\infty) f \exp \left(-\frac{E}{R\Theta}\right)
\]

Therefore on integrating,

\[
\ln \left(\frac{\rho - \rho_\infty}{\rho_\infty - \rho_\infty}\right) = - \int_0^t f \exp \left(-\frac{E}{R\Theta}\right) dt
\]

Substituting into equation (14) gives

\[
\dot{m}' = (\rho_\infty - \rho_\infty) f \exp \left(-\frac{E}{R\Theta}\right) \exp \left\{ -\int_0^t f \exp \left(\frac{-E}{R\Theta}\right) dt \right\}
\]

(15)

Integrating equation (15) with respect to \( x \) and substituting \( \rho_\infty - \rho_\infty = w_o \rho_0 \)

where \( w_o \) is the initial proportion of volatiles in the wood, gives the rate of evolution of volatiles per unit area, \( \dot{m}' \), assuming they escape as they are formed, to be

\[
\dot{m}' = \rho_0 w_o \int_0^x f \exp \left(-\frac{E}{R\Theta}\right) \exp \left\{ -\int_0^t f \exp \left(\frac{-E}{R\Theta}\right) dt \right\} dx
\]

(16)

\( \dot{m}' \) will be calculated from this equation after the temperature distribution in the wood has been evaluated below.

3.1. Semi-infinite solid neglecting cooling.

For a semi-infinite solid with a constant rate of heating at the surface, ignoring cooling from the surface, convective heat transfer due to the flow of volatiles within the solid, and self heating:

\[
\Theta = \Theta_0 + \frac{Q}{K} \left(\frac{4kt}{\pi}\right)^{\frac{1}{2}} \Gamma \frac{1}{2} \text{ierfc} \left(\frac{x}{(4kt)^{\frac{1}{2}}}\right)
\]

(17)
Multiplying by $R/E$ gives

$$R\Theta/E = R\Theta_0/E + T^{1/2} \pi^{1/2} \text{erfc} \frac{X}{T^{1/2}}$$

(18)

where $T$, $X$ are now defined by

$$T = \frac{4kT}{\pi} (\frac{r IR}{EK})^2$$

(19)

$$X = \frac{X}{\pi} \frac{r IR}{EK}$$

(20)

From equation (16):

$$\hat{a}'' = \frac{4k}{\pi^{1/2}} \frac{r IR}{EK} \Theta_0 W_0$$

$$x \int_{-\infty}^{\infty} F \exp \frac{-E}{R\Theta} \exp \left(-\int_{0}^{T} F \exp \frac{-E}{R\Theta} dT\right) dx$$

(21)

where $F = \frac{4k}{\pi} f (\frac{EK}{r IR})^2$

$E/R\Theta$ is given by equation (18)

From equation (14)

$$E/R\Theta = f_1 (T, R\Theta_0/E, X)$$

Therefore, from equation (21),

$$M = \frac{\hat{a}'' \pi^{1/2}}{4k\Theta_0 W_0} \frac{EK}{r IR}$$

$$= f_2(F, T, R\Theta/E)$$

(22)

since $X$ is a dummy variable.

Values of $M$ versus $T$ were worked out by computer for three values

of $F$ and an arbitrary value of $R\Theta_0/E$, and are shown in Fig (3). $M$ is

initially very low, rises sharply to a maximum, and then slowly decreases

as a result of depletion of volatiles near the surface. At higher values

of $F$ the maximum value of $M$ occurs at lower values of $T$. Otherwise the

values obtained vary comparatively little with $F$.  
3.2. Semi-infinite solid with cooling

The variation of the rate of volatile emission \( \dot{m}'' \), allowing for cooling at the surface, is complicated and difficult to represent comprehensively owing to the comparatively large number of quantities on which it depends. It is of interest, however, to know with which quantities \( \dot{m}'' \) can be expected to vary in order to know how it is likely to behave and to be able to obtain correlations. Three formulae are derived below giving dimensionless quantities as functions of which \( \dot{m}'' \) can be expressed.

At a depth \( x \) below the surface equation (4) becomes

\[
\dot{m}'' = \frac{H}{\varepsilon} - (\theta_s - \theta_o) - \sigma (\theta_s^4 - \theta_o^4)
\]

Substitution into equation (22) gives

\[
\frac{\dot{m}''}{\varepsilon \frac{E}{\rho_0 w_0} \frac{E}{\rho}} = f_4(\frac{Lk}{\varepsilon} (\varepsilon \frac{E}{\rho})^2, \frac{\pi k}{L} (\frac{E}{\rho} \frac{E}{\rho})^2, \frac{R \theta_o}{E}, \frac{HE}{\rho \frac{E}{\rho} \frac{E}{\rho}})
\]

(24)

The first term in the bracket on the right hand side gives the variation with \( t \). This term gives also most of the variation with \( I \).

Taking this term only gives

\[
\frac{\dot{m}''}{\varepsilon \frac{E}{\rho_0 w_0} \frac{E}{\rho}} = f_5(kt (\varepsilon \frac{E}{\rho})^2)
\]

(25)

Substituting for \( \theta \) from equation (23) into equation (16) gives

\[
\frac{\dot{m}''}{\varepsilon \frac{E}{\rho_0 w_0} \frac{E}{\rho}} = f_6(\frac{kt}{\pi} \frac{\varepsilon^2 I^2}{K^2} (\frac{\sigma}{\varepsilon})^{1/2}, \frac{\pi k}{K} \frac{E}{\rho} \frac{E}{\rho} (\frac{\sigma}{\varepsilon})^{1/2}, \theta_0 (\frac{\sigma}{\varepsilon})^{1/4}, \frac{H}{E} (\frac{\sigma}{\varepsilon})^{1/4})
\]

(26)
Equations (24), (26) and (27) represent three equations, of six dimensionless variables, for $\hat{m}''$. These equations can be derived from each other.

If $f$, $E$, $\theta_0$ and $H$ are constant then equations (24), (26) and (27) give

$$\frac{\hat{m}''}{\rho_0 w_0 (kr)^{1/2}} = f_8(kt, I/K, k, \varepsilon I)$$  \hspace{1cm} (28)$$

Equation (28) indicates the quantities with which $\hat{m}''$ can be expected to vary.

From equation (23)

$$\theta = f_{10}(kt, I/K, x, \varepsilon I)$$  \hspace{1cm} (29)$$

The term in $\varepsilon I$ in equations (28) and (29) is due to cooling at the surface. The term in $k$ is due to the fact that $\hat{m}'' = f_{11}(\theta, t) = f_{12}(\theta, kt, k)$.

The temperature inside the wood with a constant heat flux into the surface is given by

$$\theta - \theta_0 = (\theta_s - \theta_0) \text{erfc}(x(kt)^{-1/2}) / \text{erfc} 0$$  \hspace{1cm} (30)$$

where $\theta_s$ is the surface temperature.

In practice the net heat flux into the solid is not constant, owing to cooling at the surface. In this case $t$ is an effective time.

$$\hat{m}'' = \rho f_{13}(\theta, t)$$  \hspace{1cm} (31)$$

Therefore, from equation (30)

$$\hat{m}'' = \rho f_{14}(\theta_s, \theta_0, kt, k)$$  \hspace{1cm} (32)$$
Although these equations will not be used in detail the identification in the functional equations of certain variables will be used in choosing terms for the statistical analysis below.

3.3. Pseudo-activation energy

Volatile emission has a pseudo-activation energy with respect to surface temperature of approximately $13,800 \text{kJ/mol}$. This quantity is not the activation energy of the pyrolysis reaction, which occurs within the wood, where the temperature is lower than at the surface.

This value for the pseudo-activation energy is very low for a chemically based rate controlling process. It is more typical of physical processes and is probably determined by the rate of heat transfer through the solid to the pyrolysis zone.

The pyrolysis can be taken to occur at an approximately constant temperature, the char base temperature, $\theta_{ch}$. In this case the rate of emission of volatiles is proportional to the velocity, $v$, of the char zone base. The pseudo-activation energy is then given by

$$
\dot{m}'' \propto v \exp \left( -\frac{E}{R T_s} \right)
$$

Therefore

$$
\frac{1}{\dot{m}''} \left( \frac{\dot{m}''}{\dot{E}} \right)_t = \frac{1}{v} \left( \frac{\dot{m}''}{\dot{E}} \right)_t = \frac{E}{R T_s} \tag{33}
$$

where $E$ is the pseudo-activation energy calculated from the surface temperature.

Owing to cooling at the surface the net rate of heating decreases. The heating can be taken to be at a constant rate or to occur at the beginning of the period. The first approximation is represented by equation (30) and the second by the equation

$$
\theta - \theta_o = (\theta_s - \theta_o) \exp \left( -\frac{x^2}{4kt} \right) \tag{34}
$$

Equations (30) and (33) give

$$
E = \frac{R}{a} \left( \theta_{ch} - \theta_o \right) \frac{\text{ierfc} \theta}{\text{erfc} a} \left( \frac{\theta}{\theta_s - \theta_o} \right)^2
$$

where $a = x (kt)^{-1/2}$

$$
\text{ierfc} a = \text{ierfc} 0. \left( \theta_{ch} - \theta_o \right)/\left( \theta_s - \theta_o \right)
$$
erfc a ≥ 2 ierfc a, which gives

\[ E = \frac{R_s^2}{2a} \frac{\theta_s}{\theta_s - \theta_o} \]  \hspace{1cm} (35)

\[ \theta_{ch} = \theta_o + \frac{\theta_s - \theta_o}{\text{erfc} \left( \frac{R_s^2}{2E(\theta_s - \theta_o)} \right)} \]  \hspace{1cm} (36)

Equations (33) and (34) give

\[ E = \frac{1}{2} \frac{R_s^2}{\theta_s - \theta_o} \ln \frac{\theta_s - \theta_o}{\theta_{ch} - \theta_o} \]  \hspace{1cm} (37)

\[ \theta_{ch} = \theta_o + (\theta_s - \theta_o) \exp \left\{ -\frac{R_s^2}{2E(\theta_s - \theta_o)} \right\} \]  \hspace{1cm} (38)

The correct values of \( E \) and \( \theta_{ch} \) can be expected to lie between the values given by equations (35) and (36) and by equations (37) and (38). Thus if \( E \) and \( \theta_s \) are known, \( \theta_{ch} \) can be estimated, as is shown in Section 4.2. below.

4. ANALYSIS OF IGNITION DATA OF KOOHYAR

A statistical analysis was carried out on measurements by Koohyar of the rates of volatile emission, and the surface temperature of wood specimens, at ignition. The samples were subjected to irradiation from a flame since it was thought that the nature of the source might be important and irradiation by fire is the most important case for practical purposes. The experimental method used is described by Koohyar et al. In some experiments heating was from one side in others from both sides of the specimen.

In the analysis to be described below the results for oak were excluded because they appeared anomalous compared with the results for other woods. The number of sets of measurements analysed was 187 and the variables considered were chosen by reference to the formulae derived above. Where possible the results were compared with these formulae.

The variation of the rate of volatile emission with time and level of irradiance and with surface temperature was obtained. The variation of the surface temperature and the conditions for ignition were also considered.
4.1. Variation of the rate of volatile emission with time

The following results were obtained

\[ \hat{m}'' \propto (kt)^{0.40} (I/K)^{1.59} \rho^{1.81} k^{2.09} l^{-0.49} \]  \hspace{1cm} (39)

The measured variation of the initial temperature of the specimens did not have any statistically significant effect. The initial temperature was unexpectedly strongly correlated with I/K. The measurement may, perhaps as a result of time lags, have been influenced by I.

Single and double sided heating gave similar results.

The variation with k was large but not well defined, the index of k being 2.09 ± 0.70. For practical purposes k can be allowed for by including \( \rho \), which had a negative correlation with k.

The values of \( \rho \) did not vary greatly and the variation with this quantity was comparatively small and consequently was omitted from most of the analyses. Since \( \rho \) was not strongly correlated with any other quantity its inclusion or exclusion would not be expected to alter much the relationships found between the other quantities.

Omitting k and \( \rho \) gave

\[ \hat{m}'' \propto (kt)^{0.40} (I/K)^{1.48} l^{1.02} \]  \hspace{1cm} (40)

This equation shows \( \hat{m}'' \) to be proportional to \( \rho \), as indicated by equation (28). However, comparison with equation (39) indicates that the smaller variation of \( \hat{m}'' \) with \( \rho \) in equation (40) results from the exclusion of k.

Excluding \( \rho \) gave

\[ \hat{m}'' \propto (kt)^{0.25} (I/K)^{0.92} \]  \hspace{1cm} (41)

Law has found that the rate of volatile emission is approximately proportional to I. These results were not at a constant time but over a constant density loss interval. Equation (41) shows that, after a fixed time, \( \hat{m}'' \) is approximately proportional to \( (I/K) \) for woods of differing densities. If \( \hat{m}'' \) is proportional to I then the heat required to release unit mass of volatiles is constant.

The values of \( \hat{m}''/I \) at ignition were 0.244 ± 0.11 mg J^{-1}, which is equivalent to a ratio of heat input to volatile emission of 4 100 J g^{-1}. Equation (41) shows that \( \hat{m}'' \) increased with time.

Ignoring cooling, \( (\theta_s - \theta_0) \propto (I/K)(kt)^{1/2} \), from equation (1), which shows that at constant \( \theta_s \), \( kt \propto (I/K)^2 \). Equation (48) below confirms that this relationship is approximately correct at ignition. Thus excluding \( (kt)^{0.25} \) from equation (41) should reduce the index of \( (I/K) \) by 0.5. It
was found that the index was reduced by 0.57 to 0.35 ± 0.10.

A curvilinear regression gave
\[ \hat{m} \propto (kt)^{2.15} \left( \frac{I}{K} \right)^{5.08} \rho^{1.05} \exp(-0.89(I/K)(kt)^{1/2}) \] (42)

This equation represents a curve increasing with time but concave to the time axis, which shows that at ignition \( \hat{m} \) is less than the maximum value. The variation with \( kt \) and \( I/K \) is not as great as it appears in equation (42) because the exponential term decreases as the pre-exponential terms increase.

Equation (25) gives
\[ \hat{m} \propto k \rho \left( \frac{I}{K} \right)^{1.2} (kt)^{5.08} \rho^{1.05} \exp(-0.89(I/R)(kt)^{1/2}) \]

The form of this equation is in good agreement with equation (42) taking the variation of \( k \) as a variation with \( \rho \) only.

4.2. Variation of rate of volatile emission with surface temperature

The factors governing the variation of \( \hat{m} \) with \( \theta_s \) are shown by equation (32). Analysis showed that the variation with \( kt \) was small and statistically not very significant. The variation with \( \theta_s, \rho, \) and \( k \) was given by
\[ \hat{m} \propto \theta_s^{2.32} \rho^{0.97} k^{0.87} \] (43)

Analyses of surface temperature often use the temperature rise, \( \Delta \theta \), above the initial surface temperature, and on this basis the corresponding relation is
\[ \hat{m} \propto (\Delta \theta)^{1.28} \rho^{1.05} k^{0.89} \] (44)

Equations (43) and (44) show that \( \hat{m} \) is approximately proportional to \( \rho \), as is to be expected.

The index of \( k \) is equation (43) was 0.87 ± 0.064 showing that the variation with \( k \) is not significant. Excluding \( k \) gave
\[ \hat{m} \propto \theta_s^{2.27} \rho^{0.59} \] (45)

Also
\[ \hat{m} \propto \rho^{0.64} \exp(0.0034 \theta_s) \] (46)

The mean surface temperature was 637°K. The coefficient 0.0034 can, to a first order approximation, be identified with \( \frac{d}{dt} (-\frac{E}{Rt}) \) to give the
pseudo-activation energy, with respect to the surface temperature of the rate of volatile emission. The result is approximately 11,400 kJ/mol, which is similar to the value obtained by Schaffer. This quantity is given approximately by equations (35) and (37) and is not the activation energy of the pyrolysis reaction (see Section 3.3).

Substituting the mean value of the surface temperature (637°K) into equations (36) and (38) and taking θ = 335°K gives the temperature at the charring zone, θ_ch, to be between 444°K and 520°K, which is lower than the value given by Schaffer of 550°F (561°K).

4.3. Variation of surface temperature

The variation of the temperature rise at ignition, Δθ, with kt and I/K was

\[ Δθ \propto (kt)^{0.35} (I/K)^{0.98} \]  

(47)

Δθ was taken for this correlation because Δθ = 0 at t = 0.

The following relationship was also obtained:

\[ (kt/1^2) \propto \left( \frac{I}{K} \frac{1}{Δθ} \right)^{-2.39} \rho^{-0.35} \]

The index of -2.39 is in satisfactory agreement with that of -2 obtained by putting q = I in equation (1). The value of -2 is the theoretical value for short times or with no cooling.

4.4. Effect of permeability

The effect of permeability along the grain, μ, was investigated assuming the following values:

<table>
<thead>
<tr>
<th>Material</th>
<th>μ x 10^4, m^2 s^-1 atmos^-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td></td>
</tr>
<tr>
<td>Fir</td>
<td>3.6</td>
</tr>
<tr>
<td>Mahogany</td>
<td>12</td>
</tr>
<tr>
<td>Pine</td>
<td>230</td>
</tr>
<tr>
<td>Redwood</td>
<td>2.1</td>
</tr>
</tbody>
</table>

The variation of μ with was small and not statistically very significant.

4.5. Conditions for ignition

A review of ignition criteria is given by Weatherford and Sheppard. Ignition has been assumed to occur at a fixed minimum rate of volatile emission or at a fixed surface temperature. Ignition data has been correlated using calculated surface temperatures. It is to be expected that ignition would require both a minimum surface temperature and a minimum rate of volatile emission, although if one quantity reaches the minimum value first then whether ignition occurs would depend primarily on the value of the other quantity.

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The results for the variation of $\hat{m}''$ with $\Theta_s$ discussed in Section 4.2. show that an arbitrary given value of $\hat{m}''$ corresponds approximately to a fixed value of $\Theta_s$, so that in practice either a thermal model or one based on a minimum rate of volatile emission can be used to predict ignition. At ignition the values of $\Theta_s$ and $\hat{m}''$ were:

For pilot ignition:
\[ \Theta_s = 626 \pm 67^\circ\text{K} \]
\[ m'' = 5.1 \pm 1.6 \text{ g m}^{-2}\text{s}^{-1} \]

For spontaneous ignition:
\[ \Theta_s = 655 \pm 58^\circ\text{K} \]
\[ \hat{m}'' = 7.7 \pm 2.1 \text{ g m}^{-2}\text{s}^{-1} \]

The surface temperature for pilot ignition is in good agreement with the values obtained by Simms \(^{10}\) of about $350^\circ\text{C}$ ($623^\circ\text{K}$). However, the surface temperature for spontaneous ignition is much lower than the value of $525^\circ\text{C}$ ($798^\circ\text{K}$) given by Simms \(^9\). The reason for this is not clear. It may be that in the case of spontaneous ignition, but not in that of pilot ignition, the nature of the radiation source is important. The flames used by Koohyar gave a different spectral distribution from sources used previously \(^7\). It may be that spontaneous ignition is facilitated by resonant or short wave radiation from flames. Whatever the explanation the smallness of the effect of the pilot flame is remarkable.

A statistical analysis of ignition conditions by Hinkley and Thomas \(^{11}\) showed that the ignition time and surface temperature at ignition vary little with the sample thickness. Equation (44) shows that $\hat{m}''$ decreases when the sample thickness is increased. Thus it appears that ignition depends more on the surface temperature than on the rate of volatile emission.

The index of $\Theta_s$ in equation (45) was $2.27 \pm 0.26$, so that a change in $\Theta_s$ is equivalent to a proportionate change $2.27$ times greater in $\hat{m}''$. The percentage scatter of $\hat{m}''$ at ignition was on average $2.84$ times that of $\Theta_s$, which supports the theory that ignition depends more on the surface temperature. The evidence is not conclusive.

4.6. Significance of the analysis

The results analysed are not random values but values at ignition. The conclusions must therefore be treated with caution. As an example, consider the variation of $\Theta_s$ with $t$ depicted in Fig (4). $\Theta_s$ increases with $t$, the values lying about a line WXYZ, most of them falling between the lines through
The condition for ignition is represented by the line YZ, which shows the value of $\Theta_s$ at ignition decreasing with increasing $t$, as found by Simms. Ignition will occur in most cases when the values of $\Theta_s$ and $t$ lie near YZ.

Ignition can occur anywhere within a rectangle ABCD. If the variation of $\Theta_s$ with $t$ were well defined and AD, BC much smaller than AB, CD then the results would lie in a narrow rectangle about the line WX and an analysis would show $\Theta_s$ to vary with $t$ in accordance with the line WX. If, on the other hand, ignition occurred only near the line YZ, $\Theta_s$ versus $t$ being ill defined, then the results would vary in accordance with YZ. If the scatter about WX is smaller, but not much smaller, than that about YZ, then the results obtained would lie about WX but tend to be biased.

The analysis was carried out in order to compare the experimental results with the theoretical model formulated. For this purpose results for spontaneous and pilot ignition were taken together. This is to some extent justified in view of the proximity of the two ignition temperatures but it is not to be expected that the conditions at ignition would be closely determined by any ignition criterion. The results appear to be consistent with a thermal model.

5. **FLAME SPREAD**

A flame can be expected to spread to any point on a surface when the ignition criterion is satisfied there. If heating of the surface by the flame is important then the rate of spread of the flame depends on the surface conditions, in particular the surface temperature.

In many cases, particularly for thick fuels, heating of the surface by the flame is unimportant. The rate of flame spread does not then depend on the surface conditions but is equal to the rate at which the point at which the ignition criterion is satisfied progresses. In these circumstances it is to be expected that the rate of flame spread would be inversely proportional to $(d\Theta/dx)$.

6. **CONCLUSIONS**

It has been shown that, with certain simplifying assumptions, the temperature of a heated wood surface cooling by convection and radiation can be calculated in terms of a dimensionless function of the time and the intensity of irradiation, and that assuming values for the convective heat transfer coefficient and the initial temperature, the results for a semi-infinite solid can be plotted on a graph as a family of curves. Results obtained are shown in Fig.1. It was also shown that values can be calculated for a slab by including a further dimensionless function of the thickness.
An expression for the rate of evolution of volatiles from a heated wood surface was obtained and it was shown that, while this quantity depends on several factors, it can be expressed in terms of a small number of dimensionless variables. Three sets of such variables were derived.

Equations were derived which enable the char base temperature to be estimated from the pseudo-activation energy calculated from the variation of the rate of volatile emission with the surface temperature. The char base temperature for Koohyar's data was found to be between 444 and 520°K.

The analysis of Koohyar's data indicates that the variation of surface temperature and the rate of volatile emission are consistent with a thermal model and that there is a large scatter in the conditions at ignition. The rate of volatile emission was found to depend primarily on the surface temperature, so that a minimum rate of volatile emission can be used as a ignition criterion, although the surface temperature appeared to be more important. The values at ignition were:

\[ \begin{align*}
\theta_s &= 626 \pm 67^\circ K \\
\hat{m}^n &= 5.1 \pm 1.6 \text{ g m}^{-2} \text{s}^{-1}
\end{align*} \]  

for pilot ignition

and

\[ \begin{align*}
\theta_s &= 655 \pm 56^\circ K \\
\hat{m}^n &= 7.7 \pm 2.1 \text{ g m}^{-2} \text{s}^{-1}
\end{align*} \]  

for spontaneous ignition

7. REFERENCES

APPENDIX

Surface temperature of heated slab

Consider a slab receiving heat at one surface at a net rate given by

\[ q_1 = \varepsilon I - H(\theta_1 - \theta_o) - \varepsilon \sigma (e_1^h - e_o^h) \]  \hspace{1cm} (48)

and losing heat at the other surface at a net rate given by

\[ q_2 = -H(\theta_2 - \theta_o) - \varepsilon \sigma (e_2^h - e_o^h) \]  \hspace{1cm} (49)

where \( \theta_1, \theta_2 \) are the temperatures of the heated and unheated surfaces respectively.

The temperature at depth \( x \) after time \( t \) within a semi-infinite slab due to heat flux \( q \) at the surface for time \( dt \) is given by

\[ \theta(x,t) = \theta_o + \frac{q}{\pi k} \exp \left( \frac{x^2}{4kt} \right) \]  \hspace{1cm} (50)

When heat reaches a surface it can be considered to be reflected while the surface loses heat at a rate depending on its temperature. Equation (50) then gives

\[ \theta_1 = \theta_o + \int_0^t \frac{k}{\pi K} \exp \left( \frac{-(2n+1)^2}{4k(t-t')} \right) \, dt' \]

\[ \theta_2 = \theta_o + \int_0^t \frac{k}{\pi K} \exp \left( \frac{-(2n+1)^2}{4k(t-t')} \right) \, dt' \]

Substituting for \( q_1 \) and \( q_2 \) from equations (48) and (49) and putting

\[ L = \frac{(\varepsilon I/\pi K)(\sigma/I)^{1/4}}{1/4} \]

\[ T = \frac{(kt/\pi)(\varepsilon I/K)^2(\sigma/I)^{1/2}}{1/4} \]

\[ H' = \frac{(H/\varepsilon I)(\sigma/I)^{1/4}}{1/4} \]

gives

\[ P_1 = P_0 + \int_0^T \left( T-T' \right)^{1/2} \sum_{n=0}^\infty \left( (1 - H'(P_2-P_0) - (P_2^h-P_0^h)) \exp \left( \frac{-2nL}{T-T'} \right) \right) \, dt' \]

\[ P_2 = P_0 + \int_0^T \left( T-T' \right)^{1/2} \sum_{n=0}^\infty \left( (1 - H'(P_2-P_0) - (P_2^h-P_0^h)) \exp \left( \frac{-2nL}{T-T'} \right) \right) \, dt' \]

Equations (51) and (52) are simultaneous integral equations from which \( P_1, P_2 \) can be obtained.

\[ L = \frac{(H/\pi^{1/2}K)(1/\varepsilon H')}{1/4} \]

Thus if \( H, K, \theta_o, \) and 1 are known \( P_0 \) (from equation(8)) and \( L \) are functions of \( H' \). \( P_1 \) and \( P_2 \) can then be expressed as functions of \( T \) and \( H' \) only, as in the case of \( P \) for a semi-infinite solid.
$H' = 0.10$
$H' = 0.37$
$H' = 1.4$

$P_s = \theta (\frac{\xi}{r})^{\frac{3}{2}}$

$H' = \frac{H}{\varepsilon I} \left( \frac{\sigma}{I} \right)^{\frac{1}{2}}$

$\theta_0 (\frac{\sigma}{H})^{\frac{1}{2}} = 0.35$

$\tau^{\frac{1}{2}} = (\frac{4ki}{n})^{\frac{1}{2}} \frac{\varepsilon I}{k} \left( \frac{\sigma}{I} \right)^{\frac{1}{2}}$

**FIG.1. CALCULATED DIMENSIONLESS TEMPERATURE VERSUS (DIMENSIONLESS TIME)$$^{\frac{1}{2}}$$**
FIG. 2. DIMENSIONLESS TEMPERATURE VERSUS (DIMENSIONLESS TIME)_{1/2}

\[ \theta' = \frac{H}{a_l} (\frac{\varepsilon}{l})^{-\frac{1}{2}} \]

\[ \theta_0 \left( \frac{\varepsilon \varepsilon}{H} \right)^{\frac{1}{2}} = 0.35 \]
FIG. 3 VARIATION OF RATE OF VOLATILE EMISSION

\[ \frac{4\pi k t}{\pi} \left( \frac{E}{kT} \right)^2 \]

\[ R_{\theta_e}/E = 0.017 \]

1. \( \ln F = 27.4 \)
2. \( \ln F = 28.4 \)
3. \( \ln F = 31.4 \)

Where \( F = \pi f \left( \frac{E}{kT} \right)^2 \)
FIG. 4. $\theta _s$ AS A FUNCTION OF $t$