CONFIGURATION FACTOR FOR A TRIANGULAR RADIATOR

by

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SUMMARY

Equations for the configuration factor of an elemental receiver on a plane either parallel or perpendicular to a triangular radiating area are given. Values for isosceles triangles, useful for calculating heat transfer from flames, have been computed.

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1. Introduction

This note gives equations for the configuration factor, \( \phi \), from a triangular radiator to an elemental area receiver, \( ds \), on either a parallel or perpendicular plane. Values for an isosceles triangle, useful for calculating heat transfer by radiation from flames to exposed surfaces, are given in Figs 1 and 2. The theoretical basis is given elsewhere.

2. Receiver on parallel plane.

(i) Right angled triangle

Facing right angle

\[
\phi = \frac{kA}{2 \pi \sqrt{1+k^2(1+A^2)}} \left[ \tan^{-1} \frac{A}{\sqrt{1+k^2(1+A^2)}} + \tan^{-1} \frac{k^2 A}{\sqrt{1+k^2(1+A^2)}} \right]
\]

or

\[
\phi = \frac{AB}{2 \pi \sqrt{A^2+B^2(1+A^2)}} \left[ \tan^{-1} \frac{A^2}{\sqrt{A^2+B^2(1+A^2)}} + \tan^{-1} \frac{B^2}{\sqrt{A^2+B^2(1+A^2)}} \right]
\]
Right angle triangle continued

\[ \frac{a_1}{c} = A_1 \]
\[ \frac{a_2}{c} = A_2 \]
\[ \frac{b}{c} = B \]
\[ \tan \lambda = k \]

Facing base

\[ \phi = \frac{1}{2 \pi} \left[ \frac{k A_2}{\sqrt{1+k^2 (1+A_2^2)}} \left\{ \tan^{-1} \frac{A_2}{\sqrt{1+k^2 (1+A_2^2)}} + \tan^{-1} \frac{A_1 + k^2 (A_1 + A_2)}{\sqrt{1+k^2 (1+A_2^2)}} \right\} \right. \]
\[ \left. + \frac{A_1}{\sqrt{1+A_2^2}} \tan^{-1} \frac{k (A_1 + A_2)}{\sqrt{1+A_2^2}} \right] \]

or \[ \phi = \frac{1}{2 \pi} \left[ \frac{B A_2}{\sqrt{(A_1 + A_2)^2 + B^2 (1+A_2^2)}} \left\{ \tan^{-1} \frac{A_2 (A_1 + A_2)}{\sqrt{(A_1 + A_2)^2 + B^2 (1+A_2^2)}} \right. \right. \]
\[ \left. \left. + \tan^{-1} \frac{A_1 (A_1 + A_2) + B^2}{\sqrt{(A_1 + A_2)^2 + B^2 (1+A_2^2)}} \right\} + \frac{A_1}{\sqrt{1+A_2^2}} \tan^{-1} \frac{B}{\sqrt{1+A_2^2}} \right] \]

(ii) Acute angled triangle

\[ \tan \theta = K \]
\[ \beta = \frac{KAB}{2\pi \sqrt{(A\sqrt{1+K^2-B})^2 + K^2B^2(1+A^2)}} \left[ \tan^{-1} \frac{A(A + 1 + K^2 - B)}{\sqrt{(A\sqrt{1+K^2-B})^2 + K^2B^2(1+A^2)}} + \frac{\tan^{-1} B(B \sqrt{1+K^2-A})}{\sqrt{(A\sqrt{1+K^2-B})^2 + K^2B^2(1+A^2)}} \right] \] 

By employing the additive property of configuration factors, equation 4 can be used for any triangle and any position of the receiver.

(iii) Isosceles triangle

The maximum configuration factor at a given distance is for a receiver opposite the centre of gravity.

\[ \frac{h}{c} = h \]
\[ \frac{d}{c} = D \]
\[ \frac{h}{d} = f \]

From equation 3

\[ \beta = \frac{1}{\pi} \left[ \frac{fD}{\sqrt{f^2D^2+9}} \tan^{-1} \frac{1.5D}{\sqrt{f^2D^2+9}} + \frac{fD}{\sqrt{f^2(D^2+9) + 2.25}} \left\{ \tan^{-1} \frac{2f^2D}{\sqrt{f^2(D^2+9) + 2.25}} + \frac{\tan^{-1} D(f^2+0.75)}{\sqrt{f^2(D^2+9) + 2.25}} \right\} \right] \] 

or

\[ \beta = \frac{1}{\pi} \left[ \frac{H}{\sqrt{H^2+9}} \tan^{-1} \frac{1.5D}{\sqrt{H^2+9}} + \frac{HD}{\sqrt{9H^2+H^2D^2+2.25}} \left\{ \tan^{-1} \frac{2H^2}{\sqrt{9H^2+H^2D^2+2.25}} + \frac{\tan^{-1} H^2+0.75D^2}{\sqrt{9H^2+H^2D^2+2.25}} \right\} \right] \]

The solution of equation 5 is shown in Fig. 1 for different values of \( f = \frac{h}{d} \)
3. Receiver on perpendicular plane

Some of the solutions have already been given by Hamilton and Morgan; they are reproduced here for the sake of completeness.

(i) Right angle triangle

Values are given by Hamilton and Morgan

\[
\begin{align*}
\frac{a}{c} &= A \\
\frac{b}{c} &= B \\
\tan \lambda &= k
\end{align*}
\]

Facing right angle

\[
\phi = \frac{1}{2\pi} \left[ \tan^{-1} A - \frac{1}{\sqrt{1+k^2(1+A^2)}} \left\{ \tan^{-1} \frac{A}{\sqrt{1+k^2(1+A^2)}} + \tan^{-1} \frac{k^2 A}{\sqrt{1+k^2(1+A^2)}} \right\} \right] 
\]

or

\[
\phi = \frac{1}{2\pi} \left[ \tan^{-1} A - \frac{A}{\sqrt{A^2+B^2(1+A^2)}} \left\{ \tan^{-1} \frac{A^2}{\sqrt{A^2+B^2(1+A^2)}} + \tan^{-1} \frac{B^2}{\sqrt{A^2+B^2(1+A^2)}} \right\} \right] 
\]

Values are given by Hamilton and Morgan

Facing acute angle

\[
\phi = \frac{1}{2\pi} \left[ \tan^{-1} A - \frac{1}{\sqrt{1+k^2}} \right] 
\]

or

\[
\phi = \frac{1}{2\pi} \left[ \tan^{-1} A - \frac{A}{\sqrt{A^2+B^2}} \tan^{-1} \frac{1}{\sqrt{A^2+B^2}} \right] 
\]

Values are given by Hamilton and Morgan
Right angle triangle continued

Facing base

\[ \beta = \frac{1}{2 \pi} \left[ \tan^{-1} A_1 + \tan^{-1} A_2 - \frac{1}{\sqrt{1+k^2(1+A_2^2)}} \left\{ \tan^{-1} A_2 \right\} + \tan^{-1} \frac{k^2(A_1+A_2) + A_1}{\sqrt{1+k^2(1+A_2^2)}} \left\{ \tan^{-1} \frac{A_2(A_1+A_2)}{(A_1^2+A_2^2) + B^2(1+A_2^2)} \right\} \right] \]

or

\[ \beta = \frac{1}{2 \pi} \left[ \tan^{-1} A_1 + \tan^{-1} A_2 - \frac{A_1 + A_2}{\sqrt{(A_1+A_2)^2+B^2(1+A_2^2)}} \left\{ \tan^{-1} \frac{A_2(A_1+A_2)}{(A_1^2+A_2^2) + B^2(1+A_2^2)} \right\} + \tan^{-1} \frac{B^2+A_1(A_1+A_2)}{\sqrt{(A_1^2+A_2^2) + B^2(1+A_2^2)}} \right] \]
(ii) Acute angled triangle

\[
\phi = \frac{1}{2\pi} \left[ \tan^{-1} \frac{a}{c} - \frac{1}{\sqrt{1+K^2}} \tan^{-1} \frac{b}{c} - \frac{Z}{\sqrt{Z^2+2B^2K^2(1+A^2)}} \right] \\
\tan \Theta = K
\]

where \( Z = A \sqrt{1+K^2} - B \)

(iii) Isosceles triangle

The maximum configuration factor at a given distance is for a receiver opposite the centre of the base.

\[
d = D \\
h = H \\
\frac{h}{d} = f
\]

From equation (6)

\[
\phi = \frac{1}{\pi} \left[ \tan^{-1} \frac{D}{2} - \frac{1}{\sqrt{1+f^2(D^2+h^2)}} \right] \left\{ \tan^{-1} \frac{O_5D}{\sqrt{1+f^2(D^2+h^2)}} + \tan^{-1} \frac{2f^2D}{\sqrt{1+f^2(D^2+h^2)}} \right\}
\]

or

\[
\phi = \frac{1}{\pi} \left[ \tan^{-1} \frac{D}{2} - \frac{D}{\sqrt{D^2+H^2(D^2+h^2)}} \right] \left\{ \tan^{-1} \frac{O_5D^2}{\sqrt{D^2+H^2(D^2+h^2)}} + \tan^{-1} \frac{2H^2}{\sqrt{D^2+H^2(D^2+h^2)}} \right\}
\]

The solution of equation (10) is shown in Fig. 2 for different values of \( f = \frac{h}{d} \)

References


FIG. 1. ISOSCELES TRIANGLE AND PARALLEL RECEIVER
(EQUATION 5)
FIG. 2. ISOSCELES TRIANGLE AND PERPENDICULAR RECEIVER (EQUATION 10)