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THE TEMPERATURE DISTRIBUTION IN A SHEET COMPUTED BY THE ELECTRICAL ANALOGUE OF HEAT CONDUCTION

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Introduction

A quasi-infinite sheet is to be heated suddenly on one face, the other face being in contact with still air. It is desired to predict, by using the electrical analogue of heat conduction, the temperature within the sheet at various points at various times.

Properties of the sheet and temperature conditions

The sheet has the following properties:

- thermal conductivity = 2.1 E.T,U. in.ft²hr⁻¹°C⁻¹
  = 7.224 x 10⁻⁴ cal.cm⁻¹sec⁻¹°C⁻¹
- density = 1.85 gm/cm³
- specific heat = 0.235

It may be of the following thicknesses:

- Case I: Thickness = 0.360 in. = 0.914 cm
- Case II: Thickness = 0.090 in. = 0.229 cm
- Case III: Thickness = 0.180 in. = 0.457 cm

The front face is subjected suddenly to temperatures of 175°C, 234°C or 257°C for periods of 5, 10, 20 or 30 minutes. At the end of any of these time intervals the front face temperature is raised suddenly in about 3 sec. from any of the previous levels to any of the following temperatures: 256°C, 332°C, 386°C, 358°C, 455°C or 531°C. The temperature of the still air in contact with the unexposed face is taken as 30°C. It can be assumed that the heat loss from the unexposed face is Newtonian.

It is desired to know the temperatures at three points in the sheet which lie 1/4, 1/2 and 3/4 of the sheet depth from the front surface.

Method of computation

If a step function temperature rise is imposed on the heated surface of the sheet, the resulting time temperature curve at any other point in the sheet can be predicted by means of the electrical analogue of heat conduction.

The time-temperature curve to be expected from the imposition of two successive step-function temperature rises \( V_1 \) and \( V_2 \), separated by a time interval \( \tau \) is then given by Duhamel's theorem as

\[
v = f(V_1, t) + f(V_2 - V_1, t - \tau)
\]

where \( v \) is the temperature rise at the point considered, and \( f(V, t) \) is the solution to the problem in which the impressed temperature rise is \( V \).
Results

The temperature rises at different depths of the sheet in terms of the front surface temperature rise are given in figs. 1-3 for the three thicknesses of sheet.

It is assumed that at the start of the experiment the temperature of the sheet is 30°C. Thus raising the temperature of the front surface to 175°C is interpreted by the imposition temperature rise of 145°C.

Conclusions

The temperature rise at different depths of the sheet has been computed by the electrical analogue in terms of the temperature rise at the front surface.
FIG. 1. CASE I. TEMPERATURE RISE AT DIFFERENT DEPTHS EXPRESSED AS A FRACTION OF THE APPLIED FRONT SURFACE TEMPERATURE RISE.
FIG. 2. CASE II. TEMPERATURE RISE AT DIFFERENT DEPTHS EXPRESSED AS A FRACTION OF THE APPLIED FRONT SURFACE TEMPERATURE RISE.
FIG. 3. CASE III. TEMPERATURE RISE AT DIFFERENT DEPTHS EXPRESSED AS A FRACTION OF THE APPLIED FRONT SURFACE TEMPERATURE RISE.