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THE THERMAL INSULATION AFFORDED
BY A FIRE RETARDANT COATING

by

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Summary

An expression has been derived for the interface temperature rise of a combustible material with a protective layer when exposed to heat radiation. Analysis of the results gives a guide to the physical properties of an effective fire retardant paint and the conditions under which the expression gives valid results.

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1. Introduction

The amount of heat required to ignite a combustible surface can be increased by protecting it with a fire retardant paint layer. This provides thermal insulation and thus reduces the rate of rise of temperature of the combustible surface. This temperature rise depends not only on the time of exposure and intensity of radiation but also on the physical properties of the material and the protective layer. It is the purpose of this note to derive an expression for the temperature rise at the interface of a material and its protective layer and to discuss the factors upon which it depends.

A rigorous solution of the problem would be too complex to be useful for assessing the effectiveness of a fire retardant treatment and certain limiting assumptions have therefore been made:

(1) that the composite solid formed by the material and its protective layer is semi-infinite in extent;
(2) that the physical properties of the materials used are independent of temperature; and
(3) that there is no heat loss from the irradiated surface.

These assumptions will limit the validity of the results to short times of irradiation but the solution may however be used as a guide in determining the physical properties of a protective layer which is most likely to be effective in reducing the temperature rise of the protected surface.

2. Theoretical derivation of interface temperature rise

In Fig. 1 material 2 represents the material to be protected and material 1 the protective layer which has a thickness a.

Let $\Theta_1, \Theta_2$ be the temperature rise at points $x_1, x_2$ in materials 1 and 2 respectively.
Let \( k_1, k_2 \) be the thermal conductivities of materials 1 and 2
and \( \kappa_1, \kappa_2 \) diffusivities
and \( I \) incident intensity of radiation

Then the equations to be satisfied are:

\[
\frac{\partial \theta_1}{\partial x} = \frac{1}{k_1} \frac{\partial \theta_1}{\partial t} \quad \theta_1 \leq 0 \quad \text{...........(1)}
\]

\[
\frac{\partial \theta_2}{\partial x} = \frac{1}{k_2} \frac{\partial \theta_2}{\partial t} \quad \alpha \leq x \leq \infty \quad \text{...........(2)}
\]

\[
I = -k_1 \frac{\partial \theta_1}{\partial x} \quad t > 0 \quad \text{...........(3)}
\]

Assuming no thermal resistance at the interface

\[
k_1 \frac{\partial \theta_1}{\partial x} = k_2 \frac{\partial \theta_2}{\partial x} \quad \chi = \alpha \quad \text{...........(4)}
\]

Defining \( \Theta = \int_0^t \theta e^{-\kappa \tau} d\tau \) the Laplace transform of \( \theta \) then the subsidiary equations become

\[
\frac{d^2 \Theta_1}{dx^2} = \frac{p}{k_1} \Theta_1 \quad \text{...........(14)}
\]

\[
\frac{d^2 \Theta_2}{dx^2} = \frac{p}{k_2} \Theta_2 \quad \text{...........(15)}
\]

\[
\frac{I}{p} = -\kappa \frac{d \Theta_1}{dx} \quad \text{...........(16)}
\]

\[
\kappa \frac{d \Theta_2}{dx} = \kappa \frac{d \Theta_1}{dx} \quad \text{...........(17)}
\]

\[
\Theta_1 = \Theta_2 \quad \text{...........(18)}
\]
A solution of (L1) is
\[ \Theta_1 = A_1 e^{\frac{Q_a}{\kappa}} + B_1 e^{\frac{-Q_a}{\kappa}}. \] ............... (6)

A solution of (L2) is.
\[ \Theta_2 = A_2 e^{\frac{Q_a}{\kappa}} + B_2 e^{\frac{-Q_a}{\kappa}}. \] ............... (7)

\( A_1, A_2, B_1 \) and \( B_2 \) are constants to be determined from the boundary conditions.

Since \( \Theta_2 \to 0 \) as \( x \to \infty \), \( A_2 = 0 \).

The value of the remaining constants, determined from the boundary conditions stated in (L3), (L4) and (L5), are as follows:

\[ A_1 = \frac{I e^{\frac{-Q_a}{\kappa}}}{2 \beta \left( \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right)} \quad \text{where} \quad \xi = \frac{\kappa_2 q_a}{\kappa_1 q_a}, \]

\[ B_1 = \frac{I e^{\frac{Q_a}{\kappa}} (1 + \alpha)}{2 \beta \left( \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right)} \checkmark \]

\[ B_2 = \frac{I e^{\frac{Q_a}{\kappa}}}{\beta \left( \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right)} \checkmark \]

Hence
\[ \Theta_1 = \frac{I \left[ \cos \theta_1 q_a (a - x) + \frac{\xi}{\sin \theta_1 q_a} \sin \theta_1 q_a (a - x) \right]}{\beta \left[ \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right]} \] ............... (8)

\[ \Theta_2 = \frac{I e^{\frac{Q_a}{\kappa}} (a - x)}{\beta \left[ \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right]} \] ............... (9)

The Laplace transform of the interface temperature rise \( \Theta_i \) may be found by putting \( X = \alpha \) in equations (8) or (9), whence
\[ \Theta_i = \frac{I}{\beta \left[ \kappa_2 \cos \theta_1 q_a + \kappa_1 \sin \theta_1 q_a \right]} \] ............... (10)

The inverse of (10) may be found by rewriting the equation in the exponential form whence
\[ \Theta_i = \frac{2I}{\sqrt{\beta (\kappa_2^2 + \kappa_1^2)}} e^{\frac{-Q_a}{\kappa}} \quad \text{where} \quad \beta = \frac{\kappa_2}{\kappa_1} \quad \text{and} \quad \gamma = \frac{\kappa_2}{\sqrt{\kappa_1}} + \frac{\kappa_1}{\sqrt{\kappa_2}}. \] ............... (11)

Since \( \gamma e^{\frac{-Q_a}{\kappa}} \) is less than unity equation (11) may be rewritten
\[ \Theta_i = \frac{2I \gamma}{\kappa_1 \sqrt{\beta}} e^{\frac{-Q_a}{\kappa}} \frac{\left( 1 - e^{-\gamma a} \right)}{\sqrt{\beta}}. \] ............... (12)
The inverse of this type of expression is known (1) and the interface temperature rise is given by

$$
\Theta = \frac{4T \sqrt{C}}{\sqrt{V_L} + \sqrt{V_K}} \sum_{n=0}^{\infty} (\sqrt{C/n}) \left[ \left( \frac{1}{\sqrt{V_L}} - \frac{1}{\sqrt{V_K}} \right) e^{-\sqrt{C/n}} \right] \left( \frac{C}{C_{n+1}} \right) \beta \omega \left( \frac{\tau}{\omega} \right) \frac{1}{\sqrt{C}}
$$

......(13)

where \( \beta = \frac{C}{\sqrt{V_L} C} \).

**Discussion**

Equation (13) gives the interface temperature rise may be divided into two parts, a summation term which is a function of \( \beta = \frac{C}{\sqrt{V_L} C} \) and

$$
\frac{K_L}{\sqrt{V_L}} - \frac{K_K}{\sqrt{V_K}}
$$

and another factor dependent on time and the thermal properties of the two media. It will be convenient to evaluate the summation term first and it will be noted that

$$
\beta = \frac{K_L}{\sqrt{V_L}} - \frac{K_K}{\sqrt{V_K}}
$$

may have values in the range ± 1. The curves for the summation term are shown in Fig. 2. For values of \( \beta > 0.6 \) the summation is independent of \( \beta \) since the terms \( \alpha\beta \) are negligible. Under this condition the summation is only a function of \( \beta \).

The temperature rise for various values of exposure time was then computed for a combustible material having the physical properties of wood, with a protective layer of thickness 5 x \( 10^{-2} \) cm. The temperature rise was specified per unit intensity of incident radiation since these are proportional and the range of thermal constants for the protective layer covered the range of non-metals.

The thermal constants in equation (13) appear as the conductivity and diffusivity of the materials. The temperature rise was therefore computed for various values of conductivity \( K_L \), and thermal capacity per unit volume \( \beta S \), for the protective layer. From these results a measure of the effectiveness of the protective layer was determined by the time taken for the interface temperature to rise 450°C, a value at which ignition might occur. Fig. 3 shows this protection time plotted as a function of the thermal conductivity of the layer for a range of values of \( \beta S \). The corresponding results for a range of values of conductivity are shown in Fig. 4.

Figs. 3 and 4 show that for the range of thermal constants considered, the thermal capacity per unit volume of the layer largely controls the protection time though the effect of increasing the thermal conductivity of the layer is to reduce the protection time.

The protection time will of course be dependent on the thickness of the layer. Fig. 3 shows the protection time plotted as function of the layer thickness, the layer having a thermal conductivity of 1 x \( 10^{-4} \) cal/cm°C/sec. and thermal capacity per unit volume of 0.3 cal/C/C. The slope of the graph shows that in the range of thicknesses considered the advantage gained by the addition of further coats increases slightly as the thickness of the layer increases.

Since the protection time depends on the thermal properties of the protected material some materials will require less protection than others. In the following analysis it has been assumed that the thermal properties and thickness of the protective layer remain constant while the properties of the protected material are varied.
Equation (13) shows that under these conditions the temperature rise is governed by the values of the multiplying factor

\[
\frac{4 I \sqrt{C}}{\nu + \nu} \quad \text{and} \quad \eta = \frac{\kappa_1}{\sqrt{R_1}} - \frac{\kappa_1}{\sqrt{R}} + \frac{\kappa_1}{\sqrt{R_1}}
\]

These two terms may be rewritten as

\[
\frac{4 I \sqrt{C}}{\sqrt{R_1} S_1 + \sqrt{R_1} S_1} \quad \text{and} \quad \frac{\sqrt{R_1} S_1 - \sqrt{R_1} S_1}{\sqrt{R_1} S_1 + \sqrt{R_1} S_1}
\]

Consider first the effect of increasing the value of thermal conductivity \( K_1 \). This will decrease the value of the multiplying factor and increase the value of \( \eta \) which Fig. 2 shows corresponds to a decrease in the value of the series term in Equation (13). The overall effect therefore is to reduce the temperature rise for a given time of irradiation. Since the thermal properties appear in the equation as the product \( K_1/\rho S_1 \) only, then any variation in the value of \( \rho S_1 \) will result in the same change in the value of interface temperature rise as a proportional change in the thermal conductivity.

Fig. 6 shows the protection time plotted as a function of the thermal conductivity of the protected material for a range of values of thermal capacity per unit volume for a protective layer of thickness 5 x 10^{-2} cm, thermal conductivity 2.5 x 10^{-4} cal/cm\cdot\text{sec} \cdot\text{OC}. thermal capacity per unit volume 0.1 cal/\text{OC}. It can be seen from the graph that the materials requiring least protection are those with a high thermal conductivity and high thermal capacity per unit volume.

For convenience the effect of the variables in Equation (13) on the protection time is summarised in Table 1 below.

**TABLE 1**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Effect on Protection Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 )</td>
<td>Low Increase</td>
</tr>
<tr>
<td>( \rho S_1 )</td>
<td>Low Decrease</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Thin Decrease</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>Low Decrease</td>
</tr>
<tr>
<td>( \rho S_2 )</td>
<td>Low Decrease</td>
</tr>
</tbody>
</table>
4. The effect of heat loss

In the foregoing arguments heat loss from the irradiated surface has been neglected. For a semi-infinite solid receiving radiation the temperature rise of the surface is independent of the heat loss provided

\[ \psi \sqrt{\frac{C}{k^\rho \sigma}} < 0.12 \]

where \( \psi \) is the heat lost by unit area of the surface in unit time for each degree rise in temperature. A similar condition probably applies to a composite solid. A mean value of \( \psi \) for a temperature rise in the range 450°C-550°C is \( 1.4 \times 10^{-3} \text{cal/cm}^2\text{sec/°C} \). Hence the time of irradiation for which Equation (13) is valid is given by

\[ 0.12 = 1.4 \times 10^{-3} \sqrt{\frac{C}{k^\rho \sigma}} \]

or

\[ t = 73 \frac{K}{\rho \sigma} \times 10^{-2} \]

Table 2 shows the maximum time of irradiation for which heat loss can be neglected for a range of values of

<table>
<thead>
<tr>
<th>( \frac{K}{\rho \sigma} )</th>
<th>Maximum Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \times 10^{-6}</td>
<td>0.0073</td>
</tr>
<tr>
<td>5 \times 10^{-6}</td>
<td>0.036</td>
</tr>
<tr>
<td>1 \times 10^{-5}</td>
<td>0.073</td>
</tr>
<tr>
<td>5 \times 10^{-5}</td>
<td>0.36</td>
</tr>
<tr>
<td>1 \times 10^{-4}</td>
<td>0.73</td>
</tr>
<tr>
<td>5 \times 10^{-4}</td>
<td>3.6</td>
</tr>
<tr>
<td>1 \times 10^{-3}</td>
<td>7.3</td>
</tr>
<tr>
<td>5 \times 10^{-3}</td>
<td>17</td>
</tr>
</tbody>
</table>

The results in Table 2 show that in general Equation (13) will only give a correct value for the interface temperature rise for high intensities of irradiation corresponding to short exposure times.

5. Conclusions

An expression has been derived for the temperature rise at the interface of a composite solid with radiation at the surface. Analysis of the expression shows that the most effective fire retardant paint is one with a high thermal capacity and low thermal conductivity, the former condition being the more important. The protection afforded by the paint increases with the thickness of the layer. Materials which require the least protection are those with a high thermal capacity per unit volume and high thermal conductivity.

The validity of the results is limited to short periods of irradiation since heat loss from the irradiated surface was neglected. However the protection times computed from the expression will be shorter than would be obtained in practice and the expression may be used as a guide to the physical properties required of a fire retardant paint.
References


FIG. 2.

VALUE OF \[ \sum_{n=0}^{\infty} (y)^n \left[ \frac{1}{\sqrt{\pi}} \right] \frac{-(2n+1)\beta}{(2n+1)^\beta \text{erfc}(2n+1)\beta} \]

PLOTTED AGAINST \( \beta \)
FIG. 3. VARIATION IN PROTECTION TIME WITH THERMAL CONDUCTIVITY.
FIG. 4. VARIATION IN PROTECTION TIME WITH THERMAL CAPACITY PER UNIT VOLUME.
FIG. 5. VARIATION IN PROTECTION TIME WITH THICKNESS.
FIG. 6. VARIATION IN PROTECTION TIME WITH THERMAL CONDUCTIVITY OF PROTECTED MATERIAL.

Thickness of protective layer
\[ = 5 \times 10^{-2} \text{ cm.} \]