# Study on the Numerical Accuracy for the CFD

**T.Yamanashi [1], H.Uchida [2], and M.Morita [2]**

[1] *Department of Mathematics, Master's Research Course of Faculty of Science, Tokyo University of Science,1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162-8601, Japan*
[2] *Department of Mathematical Information Science, Tokyo University of Science, 1-3 Kagurazaka, Shinjuku-ku, Tokyo, 162-8601, Japan*

## Abstract

In numerical analysis for the fire mathematical models, the numerical spurious oscillations appear in the results which are computed by most of the traditional numerical methods such as the first order accuracy upwind difference scheme and the third order accuracy upwind difference scheme. The new numerical scheme is applied to PDEs with convection terms in the fire mathematical models. This scheme is based on switching high accurate interpolation of the profile to one satisfying the TVD condition for space variable over a local area when the numerical spurious oscillations is appeared.
In this paper, we propose the new scheme, TVDCIP(Total Variation Diminishing Cubic Interpolated Propagation), that consists of the spatial interpolation and the mathematical algorithm for switching scheme with high order accuracy. TVDCIP method is applied to solve the nonlinear hyperbolic partial differential equations numerically such as Navier-Stoke's type equations.

## 1. Introduction

Most of fire phenomena in fire science and technology are modeled numerically by the partial difference equations (PDEs) in space and time.

In computational fluid dynamics, the convection equations are formulated as one of the partial differential equations. Normally, the convection terms in these equations cause the numerical spurious oscillations.

In the finite difference method, it is necessary to obtain the high accuracy numerical results of PDEs. However, in the high-order accuracy scheme, the numerical spurious oscillations might be appeared and influence the numerical stability in the system. In order to clear the numerical spurious oscillations and the numerical viscosities of the computational results,

*Corresponding Author*
*E-mail address*: dr_244jp@yahoo.co.jp

we introduce the following linear convection equation,

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0, \quad u(x,t) \in (0,t) \times [0,L]$$

where $c$ is a nonnegative constant.

Figure 1 shows the numerical results by the first-order accuracy upwind difference scheme, the second-order accuracy central difference scheme and the third-order accuracy upwind difference scheme.

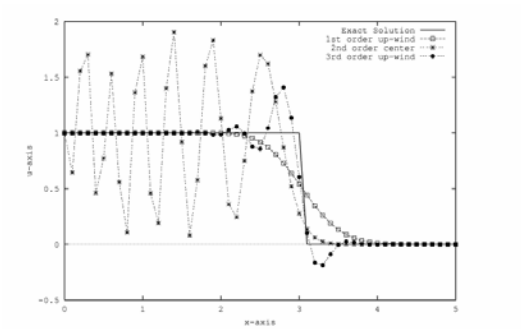


**Figure 1.** The numerical solutions by the various difference schemes and the exact solution.

In Figure 1, at the first-order accuracy upwind difference scheme, numerical viscosities are strongly appeared and numerical oscillations are not. The effect of numerical viscosities will be reduced by the high-order accuracy difference scheme. On the other hand, the numerical oscillations are appeared to the second-order accuracy central difference scheme and the third-order accuracy upwind difference scheme in these results. The relationship between the reduction of numerical viscosities and the suppression of numerical spurious oscillations is exclusive. The Total Variation Diminishing (TVD) method [1] will be cleared above the problems. The TVD method has successfully obtained the adequate results for the compressible viscous fluid dynamics problems in fire phenomena. It is necessary to keep the conservation form mathematically.

In this paper, we propose the new method (TVDCIP method) that has no any restrictions to the conservation form. However it is necessary to take account of the flow direction. Until now, the CIP method [2] has been presented, and applied to many problems [3,4,5]. In the CIP method, the spatial profile is described by third-order Hermite interpolation in the convection terms and updated by shifting the profiles according to the local exact solution. The CIP method for non-conservative form has been developed, and the computed results have been comparable good agreement with the conservative schemes such as the TVD method.

## 2. Numerical Scheme

### 2.1 mCIP method

The mCIP method is defined as follows and the mathematical algorithm of the mCIP method [6] is as follows.

#### 2.1.1 Definition of mCIP method

The mCIP method is defined by switching the cubic interpolation of the profile having extremum to the first-order accuracy for space variable over a local area in the CIP method.

The mCIP method is introduced to the following interpolating function,

$$F_j(x) = a_j(x-x_j)^3 + b_j(x-x_j)^2 + c_j(x-x_j) + d_j$$

where $a_j, b_j, c_j$ and $d_j$ are defined as follows respectively.

$$a_j = \frac{f_j'^n + f_{j-1}'^n}{(\Delta x)^2} - \frac{2(f_j'^n - f_{j-1}'^n)}{(\Delta x)^3}$$

$$b_j = \frac{3(f_j'^n + f_{j-1}'^n)}{(\Delta x)^2} + \frac{2 f_j'^n + f_{j-1}'^n}{(\Delta x)}$$

$$c_j = f_j'^n$$

$$d_j = f_j^n$$

#### 2.1.2 Flow Diagram of the mCIP method

**Step 0** : Set each parameters.

- $g(x) = F'(x)$
- $exremum = g(-b_j / 3a_j)$
- $bound\,1 = g(x_{j-1}), bound\,2 = g(x_j)$

**Step 1** : $a_j \neq 0$

**Step 2** : $x_{j-1} \leq x_j - b_j / 3a_j \leq x_j$

**Step 3-1** : extremum * bound1 $\geq$ 0 and extremum * bound2 $\geq$ 0

**Step 3-2** : bound1* bound2 $\geq$ 0

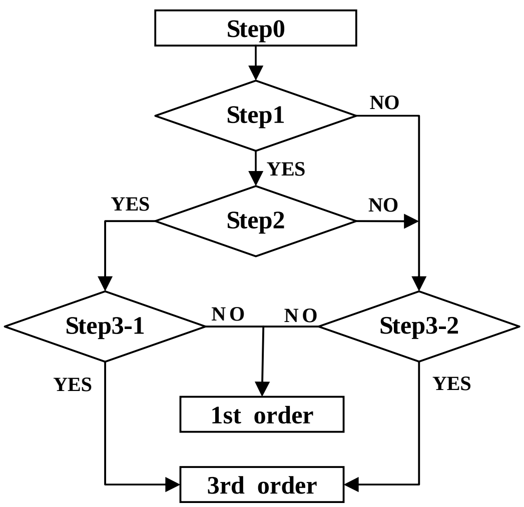


**Figure 2.** Flow diagram of the mCIP method.

### 2.2 TVDCIP method

The TVDCIP method is defined and the mathematical algorithm for the TVDCIP method is follows.

#### 2.2.1 Definition of mCIP method

The TVDCIP method is defined by switching the cubic interpolation of the profile having extremum to one satisfying the TVD condition for space variable over a local area in the CIP method.

#### 2.2.2 Flow Diagram of the TVDCIP method

The flow diagram of the TVDCIP method is the same as that for mCIP if we change the first-order accuracy interpolation to the third-order accuracy satisfying TVD condition in Figure 2.

## 3. Numerical Experiments

There are the numerical experiments for the one-dimensional nonlinear hyperbolic Euler equations called the compressible viscous fluid flow equation in Case 1, and the two-dimensional linear convection equation in Case 2 as follows.

**Case 1**

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + u \dfrac{\partial \rho}{\partial x} = -\rho \dfrac{\partial u}{\partial x} \\ \dfrac{\partial u}{\partial t} + u \dfrac{\partial u}{\partial x} = -\dfrac{1}{p} \dfrac{\partial p}{\partial x} \\ \dfrac{\partial e}{\partial t} + u \dfrac{\partial e}{\partial x} = -\dfrac{p}{\rho} \dfrac{\partial u}{\partial x} \end{cases}$$

**(Initial conditions)**

$$u(x,0) = 0$$

$$p(x,0) = \begin{cases} 1 & (x < 100) \\ 0.1 & (x > 100) \end{cases}$$

$$\rho(x,0) = \begin{cases} 1 & (x < 100) \\ 0.125 & (x > 100) \end{cases}$$

**Case 2**

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 0$$

**(Initial conditions)**

$$f(x, y, 0) = \begin{cases} 1 & (0 < x < 1, 0 < y < 1) \\ 0 & otherwise \end{cases}$$

## 4. Computational Results

**Case 1**

The computational results of the density, the velocity and the pressure for Case 1 are obtained with Δx=1.0 and Δt=0.1 at 400 steps by the TVDCIP method, the first-order accuracy upwind difference method and the third-order accuracy upwind difference method, respectively as shown in Figure 3-5.

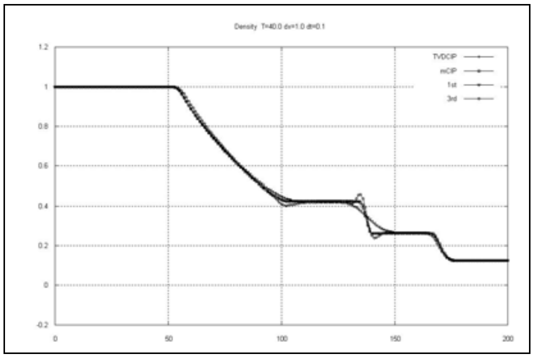

**Figure 3.** Numerical results of density

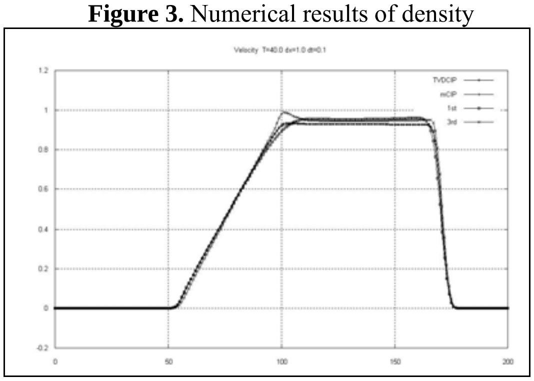

**Figure 4.** Numerical results of velocity

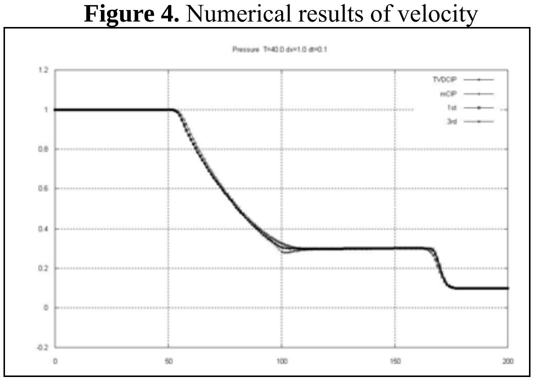

**Figure 5.** Numerical results of pressure

**Case 2**

The computational results and the exact solution for Case 2 are obtained with Δx=0.1 and Δt=0.01 at 200 steps by the TVDCIP method, the mCIP method, the first-order accuracy upwind difference method and the third-order accuracy upwind difference method, respectively as shown in Figure 6-9. The counter-maps of the values of function $f(x, y, 2)$ on the xy-plane are shown in Figures 6, 7, 8, 9, and 10.

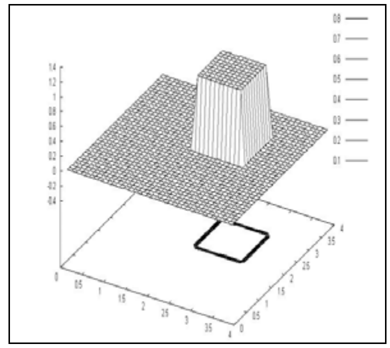
**Figure 6.** Analytical solution

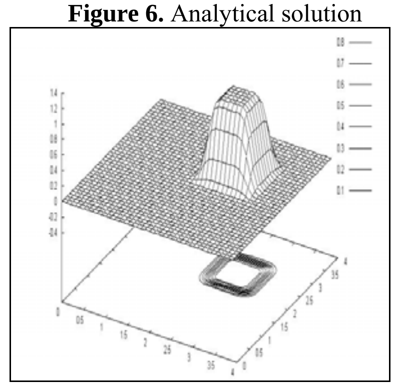
**Figure 7.** Numerical results by TVDCIP method

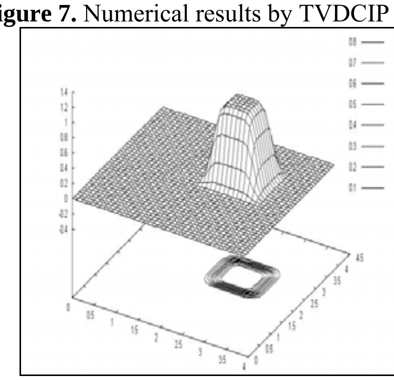
**Figure 8.** Numerical results by mCIP method

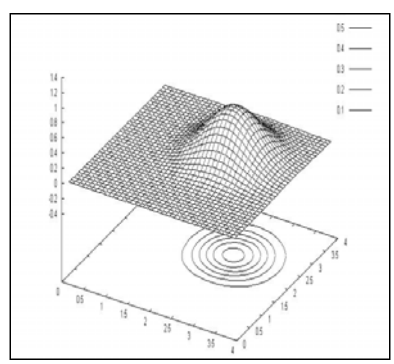
**Figure 9.** Numerical results by 1st order difference method

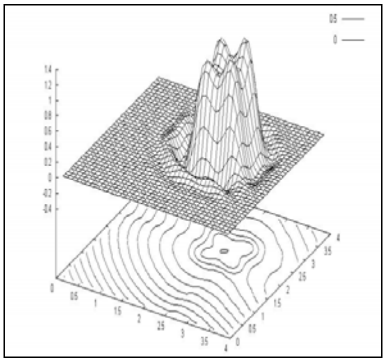

**Figure 10.** Numerical results by 3rd order difference method

## 5. Discussions

In Case 1, the computational results in Figure 3, 4 and 5 are quite difference among the scheme with the numerical viscosities, and the numerical spurious oscillations. The numerical spurious oscillations are only appeared to the third-order accuracy upwind difference method. On the other hand, the numerical viscosities are only appeared to the first-order accuracy difference method. The mCIP method and the TVDCIP method are almost no appeared both the numerical spurious oscillations and the numerical viscosities.

In Case 2, the similar computational results are obtained as Case 1. There are quite differences between the analytical results in Figure 6 and the computational results by first-order accuracy difference method in Figure 9, and also quite differences between the analytical results in Figure 6 and the computational results by third-order accuracy upwind difference method in Figure 10.

The numerical viscosities are clearly appeared in Figure 9, and the numerical spurious oscillations are also clearly appeared in Figure 10. On the other hand, the computational results by mCIP method and TVDCIP method in Figure 7 and 8 are almost agreed with the analytical results in Figure 6.

In above mentions, the TVDCIP method and the mCIP method might be good agreement for the fire simulation.

## 6. Conclusion

The TVDCIP method and the mCIP method have the good results compared with some traditional methods such as the first-order accuracy upwind difference method and third-order accuracy upwind difference method that are used in fire simulations. By numerical experiments, these methods are useful for numerical analysis for not only the non-linear hyperbolic partial differential equations but also 2-dimensional linear convection equation. The higher-dimensional problems like the Navier-Stokes type equations for the fire problem are applied in the same way.

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