Equivalent Fire Duration Based on Time- Heat Flux Area

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ABSTRACT

A simple formula was proposed to calculate the equivalent fire exposure based on time-heat flux area. Given a fire severity and duration of design fires, the heat flux absorbed by building elements was described by an analytical formula for heat conduction in a semi-infinite medium. Using the formula, the time to give the same amount of heat to building elements under design fires and under standard (ISO 834 fire) is derived. Assuming that the behavior of building elements is identical if the total amount of heat is equivalent, then the formula can be used as a representation of equivalent fire duration for structural fire resistance design. Numerical examinations and several experimental results show that the assumption holds fairly well for practical range of application.

Key words: equivalent fire duration, ISO 834 fire, design fires, actual fire, severity

INTRODUCTION

In the fire resistance tests of building construction, standard time-temperature curve is adopted in order to classify building elements into fire resistance ratings. The standard time-temperature curve is one of the representatives of actual fires. However, the actual fires differ considerably from the standard fire depending on fuel load density, internal surface area of compartment boundary and ventilation parameter. Thus various formulas have been proposed in order to correlate the behavior of specific building element under actual fires with that under standard fire.

Kawagoe made extensive calculations and experiments for reinforced concrete elements. After his results, he concluded that the maximum temperature of reinforcing steel bar...
Compartment Fire Temperature

Before we discuss the equivalent fire duration, we limit our interest to fully developed compartment fires. There are many proposals for predicting compartment fire temperatures. Among them, we adopted the following set of formula proposed by McCerffry at al.\textsuperscript{(10)} and extended by Matsuyama et al.\textsuperscript{(11)}.

Compartment fire could be classified by using the burning type factor, which relates to air supply rate per unit fuel surface area (see nomenclature for symbols),

\[
\chi = A_w \sqrt{\frac{H_w}{A_{	ext{fuel}}}},
\]

where the fuel surface area can be calculated by\textsuperscript{(12)}

\[
A_{\text{fuel}} = \phi(w_{A_w}) = 0.54w^{-0.3} \times w_{A_w} = 0.54w^{-0.3} A_w,
\]

given the fuel load density as a characteristic value for building use.

**Ventilation Controlled fires**: ($\chi \leq 0.07$)

The compartment fire temperature and duration would be

\[
T_f - T_w = 3.0T_w \left( A_w \sqrt{H_w} / A_{\text{fuel}} \sqrt{\beta \psi} \right)^{1/3} t^{1/6},
\]

\[
t_f = w_{A_w} / 0.1 A_w \sqrt{H_w}.
\]

**Fuel Surface Controlled Fires**: ($\chi > 0.07$)
McClure’s formula is applied with empirical heat release rate formula developed by Ohmiya et al.\(^{[10]}\)

\[
T_r - T_w = 0.0227\frac{Q^{1/3}}{(A_t/\sqrt{\lambda D_{Ac}})^{1/2}} t^{1/2},
\]

\(t_0 = 16000w/A_t \) \(Q\),

\[
Q/A_{net} = \begin{cases} 
112 & (x \leq 0.1) \\
192x \exp(-11x) + 48 & (0.1 \leq x)
\end{cases}
\]

**Standard Fire**

The standard fire temperature in ISO 834 can be approximated by

\[
T_{r,ISO} - T_w = 345 \log_{10}(8t/60 + 1) \approx 230t^{1/4}
\]

without considerable loss of accuracy.

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**General Expression for Compartment Fires**

All the fires defined by equations (4), (6) and (9) are expressed by \((1/6)\) power of time. Thus it is possible to write

\[
T_r - T_w = \int q(t) dt = \int \frac{3\sqrt{\pi}}{4} \sqrt{\lambda D_{Ac}} \beta t^{1/6} dt.
\]

**Heat Flux Absorbed by Building Elements**

Most fire effects are transient. Thus we could approximate most of the building elements by semi-infinite body with respect to heat conduction. If the absorbed heat flux is constant over time, an analytical expression is possible for surface temperature rise.

\[
T_r - T_w = \frac{2q}{\sqrt{\pi}} \sqrt{\lambda D_{Ac}} t^{1/2},
\]

Rearranging the terms, we get,

\[
q = \frac{\sqrt{\pi}}{2} \sqrt{\lambda D_{Ac}} (T_r - T_w).
\]

Approximating that surface temperature is close to the fire temperature, we obtain

\[
q = \frac{\sqrt{\pi}}{2} \sqrt{\lambda D_{Ac}} t \approx \frac{\sqrt{\pi}}{2} \sqrt{\lambda D_{Ac}} \beta t^{1/6}
\]

Integrating equation (13) over fire duration, we get the total amount of heat absorbed by the surface as

\[
E(t) = \frac{3\sqrt{\pi}}{4} \sqrt{\lambda D_{Ac}} \beta t^{1/6}.
\]

**Equivalent Fire Duration**

Let the fire severity coefficient be \(\beta_{\text{design}}\) and fire duration and \(t_{f,\text{design}}\) for specific design fire. Then the time-flux area will be

\[
E_{\text{design}} = \frac{3\sqrt{\pi}}{4} \sqrt{\lambda D_{Ac}} \beta_{\text{design}} t_{\text{design}}^{1/2}.
\]
For standard fire, the same relationship holds for the heating period up to $t_w$.

$$E(t_w) = \frac{3\sqrt{\pi}}{4} \sqrt{\lambda \rho c_p \times 230^2 t_w^{3/2}}.$$  \hfill (16)

Equating equation (15) and (16), we obtain,

$$t_{eq} = \left( \frac{E_{(design)}}{230} \right)^{2/3} t_{eff}.$$  \hfill (17)

**VERIFICATION BY NUMERICAL ANALYSIS**

**Methods**

For the fires described by $\beta t^{1/6}$, heat flux is not constant over time. In a precise manner, the heat flux would be given by a convolution integral of fire temperature and impulse response of building element. Thus the validity of equation (13) and (17) were checked against numerical calculations of heat conduction.

We selected a 100mm thick concrete wall (thermally thick wall) and a 20mm thick calcium silicate board (thermally thin wall). For both of them, the wall temperature and absorbed heat flux were calculated for the design fire temperatures shown in FIGURE 4 and for the standard fire. Namely, the equation of heat conduction

$$\rho c_p \frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T(x,t)}{\partial x} \right),$$  \hfill (18)

was solved by finite difference method as shown in FIGURE 5. The boundary conditions at $x = 0$ and $x = L$ are, respectively,

$$-\lambda \frac{\partial T(0,t)}{\partial x} = h_i (T_i - T(0,t)) + \varepsilon_{\alpha} \sigma (T_i^4 - T^4(0,t)),$$  \hfill (19)

$$\lambda \frac{\partial T(L,t)}{\partial x} = h_e (T_e - T(L,t)) + \varepsilon_{\alpha} \sigma (T_e^4 - T^4(L,t)).$$  \hfill (20)

Based on the calculated temperature profile, time-heat flux area was calculated by

$$E_{(design)} = \int_0^{t_{eff}} q(t)dt = \int_0^{t_{eff}} h_i (T_i - T(0,t)) + \varepsilon_{\alpha} \sigma (T_i^4 - T^4(0,t))dt$$  \hfill (21)

for design fires. For the standard fire, the same formula was applied to calculate the heating duration to apply the same time-heat flux area.

![Figure 5 One-dimensional wall heat conduction](image)

**Results**

As an example, the results for $A_w=1$ ($\beta = 181$ [K/s$^{16}$], $t_{eff}=1060$ [s]) is shown in FIGURE 6. The time-flux area for the design fire temperature was 156.6 [MJ/m$^2$]. Using the calculated time-flux relationship for the standard fire, the time to the same time-flux area was found to be 120 minutes. On the other hand, equation (17) yields,

$$t_{eq} = \left( \frac{181}{230} \right)^{2/3} \times 177 = 123,$$  \hfill (22)

which is very close to numerical results.

![Figure 6 Calculation of Equivalent Fire Duration Based on Time-Flux area (Concrete 100mm)](image)

Similar analysis was carried out for all the design fire temperatures shown in Figure 4 both for concrete and for calcium silicate board. The results are shown in Figure 7. As to thermally thick wall (concrete), the agreement is very good between numerical results and simple
formula (17). As to the thermally thin walls (calcium silicate board), the agreement is not so good as that of thermally thick walls. Simple formula tends to give smaller values. But this tendency is only for mild (small $\beta$) fires, where the problem of fire resistance is not serious. Thus it is possible to conclude that the simple formula (17) is accurate enough to calculate the equivalent fire duration for the fires described by $\beta t^\alpha$.

![Graph](image1)

FIGURE 7 Comparison of equivalent fire duration calculated by simple formula (17) with precise calculation

Wall Temperature Rise

The calculated temperature rise of the unexposed surface is summarized in FIGURE 8. As is shown in FIGURE 8a), the temperature rise greatly depends on fire severity. In FIGURE 8b), the same data are plotted versus the equivalent time $t_e = (\beta/230)^{\alpha/2} t$. The scatter during the fire period can be greatly reduced. Thus it is reasonable to scale the “real” time in accordance with equivalent time $t_e = (\beta/230)^{\alpha/2} t$ at least in the period of heating.

In FIGURE 9, maximum surface temperature for design fires are compared with those for standard fires at equivalent duration. FIGURE 9a) shows the temperature at the end of fire. In case of concrete, agreement is good for severe fires. For mild and short fires, the temperature at equivalent fire duration is very low because of the large heat capacity of concrete, which delays the heat propagation to unexposed surface. However, this type of error is not a significant problem for practical fire resistance design. In case of calcium silicate board, the agreement is poor. The temperature for standard fire is higher than the design fires for most of the cases. This side of error is always conservative. Thus the present method is acceptable as a design tool. FIGURE 9b) shows the maximum temperature including the post fire period. In case of concrete, the agreement is much better. As to the calcium silicate board, the results are still poor but conservative.

![Graph](image2)

FIGURE 8 Unexposed surface temperature of Concrete Wall (100mm) heated by $\beta t^\alpha$-fires and ISO 834 fire.

![Graph](image3)

FIGURE 9 Comparison of temperature of unexposed surface

EXPERIMENTAL SURVEY

To investigate the appropriateness of the present method, a survey was carried out to check the accuracy of proposed formula against existing fire test data.

Thermally Thick Case

As a representative for thermally thick construction, fire test data on concrete slab\(^{14}\) was analyzed. In Figure 10, three fire test data for concrete floors are plotted. The difference
between the three tests is the furnace temperature, which was altered to 90, 100 and 110% of the standard time-temperature curve. The difference in temperature rise in the three tests is greatly reduced by plotting versus equivalent time, \( t_{eq} = (\beta / 230)^{1/3} t \) instead of actual time.

![Figure 9](image)

**FIGURE 9** Between the three tests is the furnace temperature, which was altered to 90, 100 and 110% of the standard time-temperature curve. The difference in temperature rise in the three tests is greatly reduced by plotting versus equivalent time, \( t_{eq} = (\beta / 230)^{1/3} t \) instead of actual time.

**Thermally Thin Case:**

Figure 11 shows the three test results of four thermally thin wall specimens, heated by standard fires (44 and 54 minutes) and 80% of standard fire for 72 minutes. Similar to thermally thick slabs, the difference of temperature rise during heating period was reduced by using the equivalent time. However, the maximum temperature does not agree well as was predicted theoretically in Figure 9.

Figure 12 shows the test results of gypsum board (12mm +12mm). The time to integrity failure was measured at 33, 45 and 57 minutes, for 80, 100 and 120% of the standard fire. By using equation (17), those times are converted to 41 and 43 minutes, which are sufficiently close to the time under standard fire (45 minutes).

**Figure 10** Test results of 100mm thick concrete floor covered by a steel plate.

**Figure 11** Test results of FCB (fiber cement board, L=50mm, w=3.23%wt.), ALC (Autoclaved Lightweight Concrete, 50mm, 14.5%), CSB (Calcium Silicate Board, 40mm, 3.85%) and PBM (Plastic Beads Mortar, 70mm, 15.4%)

**CONCLUSIONS**

A simple formula for equivalent fire duration was proposed based on the equivalency in time-heat flux area, not by time-temperature area. The accuracy of the formula was checked against numerical calculations of the heat flux history for thermally thin and thick walls. A survey of existing experimental data was carried out to investigate the applicability of the proposed formula. In summary the formula gives reasonable results for temperature during heating period. As to the cooling period (post-fire), the temperature can not be predicted accurately, but conservative results can be obtained.
NOMENCLATURE

Alphabets
- $A_r$: internal surface area [m$^2$]
- $A_f$: floor area [m$^2$]
- $A_w$: window area [m$^2$]
- $A_s$: fuel surface area [m$^2$]
- $c$: specific heat [kJ/kg.K]
- $E$: time-heat flux area [kJ/m$^2$]
- $H_w$: window height [m]
- $w$: fuel load density per unit floor area (wood equivalent) [kg/m$^2$]
- $A_w$: ventilation factor [m$^{2.5}$]
- $q$: heat flux [kW/m$^2$]
- $Q$: heat release rate [kW]
- $t$: time [s]
- $t_e$: equivalent time [s]
- $t_r$: fire duration [s]
- $h$: convective factor [W/m$^3$.K]
- $L$: wall thickness [m]
- $T_{(x,t)}$: temp. of building element [K]
- $T_f$: fire temperature [K]
- $T_a$: ambient temperature [K]
- $\rho$: density [kg/m$^3$]
- $\lambda$: thermal conductivity [kW/m.K]
- $\sigma$: Stefan-Boltzman const. [W/m$^2$.K$^4$]
- $\rho_e$: density [m$^{2.5}$]
- $\gamma$: thermal inertia [kJ/s$^{1.5}$.m$^2$.K]
- $E_w$: effective emissivity [-]
- $\varepsilon$: effective emissivity [-]

Greek letters
- $\gamma$: specific heat [kJ/\text{kg.K}]
- $\phi$: surface area coeff. [-]

REFERENCES
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LARGE SCALE EXPERIMENTS