FLAME SPREADING OVER A ROD FUEL IN A QUIESCENT ENVIRONMENT

Xian-Ting LI and Qi-Sen YAN
Department of Thermal Engineering, Tsinghua University, Beijing, China, 100084

ABSTRACT

A theoretical model for flame spread over a rod fuel in a quiescent environment is presented that can be used to predict the structure and spreading rate of flame. The model includes steady-state, two-dimensional momentum, energy and species equations in gas phase and continuity and energy equations in solid phase.. In a reference frame attached to the flame front, the flame spreading rate becomes an eigenvalue for this problem. The relationships between flame spreading rate and diameters of fuel rod and ambient oxygen concentration are presented. It is concluded that the flame spreading rate decreases as the diameter of fuel rod is increased whereas is increased as ambient oxygen concentration is increased.

Key Words: *Quiescent environment, Flame spreading, Rod fuel*

NOMENCLATURE

As'	Pre-exponential factor for solid pyrolysis
B_F'	Frequency factor
C_p'	Gas-phase specific heat
C_{ps}'	Specific heat for solid fuel
D'	Dimensional species diffusivity
Dr'	Reference species diffusivity
E'	Activation energy for gas phase
E	Dimensionless activation energy in gas phase
Es'	Activation energy for fuel pyrolysis
Es	Dimensionless activation energy of pyrolysis
Le	Lewis number
Lr'	Reference thermal length
Lv'	Latent heat of vaporization
Lv	Dimensionless latent heat of vaporization
m_f	Fuel mass fraction
m_{ox}	Oxygen mass fraction
$m_{ox\infty}$	Ambient oxygen mass fraction
$\dot{m}_s''$	Burning rate on solid fuel
$\dot{M}_s''$	Dimensionless burning rate on solid fuel
p'	Pressure
p_∞'	Ambient pressure
p	Dimensionless pressure
Pr	Reference Prandtl number
q_{in}	Heat flux from gas phase to solid phase
r '	Co-ordinate vertical to fuel surface
r	Dimensionless co-ordinate vertical to fuel surface
R	Universal gas constant

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r_{max}	Maximum r in computational domain	u'	Velocity parallel to the fuel surface
r_0'	Radius of solid rod	u	Dimensionless velocity parallel to the fuel surface
r_0	Dimensionless radius of solid rod	v'	Velocity vertical to the fuel surface
Re	Reynolds number	v	Dimensionless velocity vertical to the fuel surface
Sc	Conduction / convective parameter	V_f'	Flame spreading rate
s	Stoichiometric oxidizer / fuel mass ratio	V_f	Dimensionless flame spreading rate
S_φ	Source term for equation for property φ	V_{ref}	Reference velocity
T'	Gas temperature	v_w	Dimensionless interfacial velocity
T	Dimensionless gas temperature	x'	Co-ordinate parallel to the fuel surface
T_f'	Adiabatic stoichiometric flame temperature or flame temperature at the foot of flame	x	Dimensionless co-ordinate parallel to the fuel surface
T_∞	Ambient temperature	x_{bo}	Dimensionless burnout location
Tr'	Reference temperature	x_{max}	Maximum x in computational domain
Ts'	Solid temperature		
Ts	Dimensionless solid temperature		

Greek Symbols

α_r'	Thermal diffusivity of the gas	μ_r'	Reference dynamic viscosity
α_s'	Thermal diffusivity of the solid	ν'	Kinematic viscosity of the gas
α_s	Dimensionless thermal diffusivity of the solid	ρ'	Gas mixture density
Γ_φ	Effective exchange coefficient for the property φ	ρ	Dimensionless gas mixture density
$\Delta Hc'$	Heat of combustion per unit mass of fuel	ρ'_{bo}	Burnout density of the solid
ΔHc	Dimensionless heat of combustion	ρ_{bo}	Dimensionless burnout density
λ'	Thermal conductivity of gas mixture	ρ_r'	Reference gas mixture density
λ_r'	Reference thermal conductivity	ρ_s'	Solid fuel density
λ_s'	Thermal conductivity of the solid	ρ_s	Dimensionless solid density
γ	Temperature ratio, Tr'/T_∞	$\rho'_{s\infty}$	Solid density in ambient condition
μ'	Dynamic viscosity	ρ_w	Gas density at interface
μ	Dimensionless dynamic viscosity	$\dot{\omega}_f$	Dimensionless reaction rate
		$\dot{\omega}_f'$	Reaction rate for fuel

Subscripts

f	Fuel, flame	w	Solid surface
g	Gas	∞	Ambient condition
ox	Oxygen	max	Maximum
r	Reference state (gas phase)	min	Minimum
s	Solid		

Superscripts

'	Dimensional quantity	"	Per unit area
•	Per unit time		

1 INTRODUCTION

The phenomenon of the propagation of flames over the surface of a combustible material has received considerable attention during the last decade since it relates to the prevention and control of fires and to the fundamental questions in combustion science. The spread of flame results from the complex interaction of transport processes in the gas and condensed phases, the vaporization of the fuel and the chemical reaction of the fuel vapors with the gaseous oxidizer.

Many investigators have studied the problem of flame spread over a thermally thin solid fuel. De Ris [1] developed the first significant model for spread over thermally thin fuels in an opposing flow that resulted in a simple predictive formula. The paper by Frey and Tien [2] was the first successful analysis to present the flame spreading rate as a function of opposed flow velocity by numerical methods. They point out the effect of finite rate chemical kinetic on the flame front although the Oseen approximation was retained. Bhattacharjee *et al.* [3] present a theoretical model that can be used to predict the structure and rate of spread of an attached diffusion flame moving over a thermally thin pyrolyzing combustible placed in a gravity-free, quiescent, oxidizing environment.

However, for the problem of flame spread over a rod fuel, few papers can be seen and more work need to be done. Sibulkin and Lee [4] measured the flame spreading velocity using PMMA cylinders as fuel rods. Weber and de Mestre [5] measured the flame spread rate on single ponderosa needles. So far there are few papers on the theoretical study of flame spreading on rod fuel.

A theoretical model for flame spread over a rod fuel in a quiescent environment is presented here and numerical solutions to the model are given. By using the model, effects of rod radius and ambient oxygen concentration on flame spreading rate are studied.

2 MATHEMATICAL MODEL

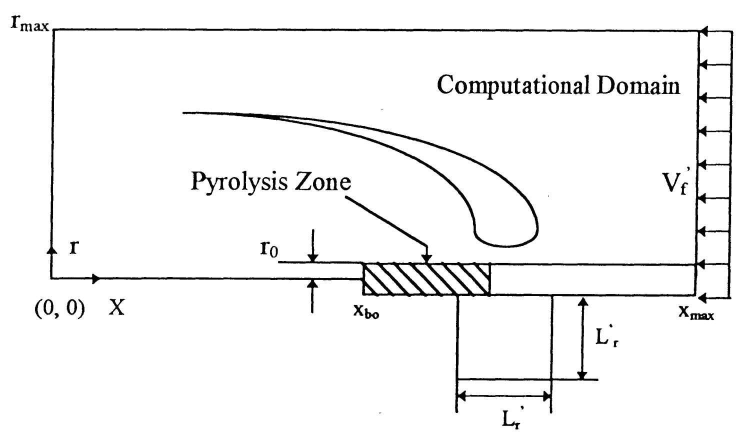

Figure 1 Schematic of the flame spread configuration and computational domain

The specific problem to be studied in this paper is steady diffusion flame spread over a solid rod fuel in a quiescent environment. In a reference frame attached to the flame front, as shown in Figure 1, the flame becomes stationary and the oxidizer environment as well as the solid fuel bed are moving into the flame with velocity V_f'. The flame spread rate is now an eigenvalue and is not known *a priori*.

The model consists of the elliptic, partial differential equations for the conservation of energy, species, mass and momentum in the gas phase and the ordinary differential equations for the conservation of mass and energy in the solid phase. Before introducing the governing equations for this combustion model, several assumptions are made. They are: (1) The flow field is quasi-steady, laminar and cylindrical axis-symmetrical, (2) Radiative heat transfer and body forces are neglected, (3) The fluid is an ideal gas mixture. (4) The gas phase chemistry is described by a one-step overall chemical reaction, which is :

$$1[F] + s[O] \rightarrow (1+s)[P] + \Delta Hc' \qquad (1)$$

where s is stoichiometric oxidizer / fuel mass ratio and $\Delta Hc'$ is the heat of combustion per unit mass of fuel. The corresponding chemical kinetics is expressed as

$$\dot{\omega}_f' = B_F' \rho'^2 m_f m_{ox} \exp(-\frac{E'}{RT'}) \qquad (2)$$

where $\dot{\omega}_f'$ is the fuel consumption rate . (5) The solid rod is thermally thin, it means the temperature distribution across it is nearly uniform and the property variations in the r-direction can be assumed negligible. (6) An Arrhenius-typed pyrolysis law is used to describe the fuel gasification:

$$\dot{m}_s'' = A_s''(\rho_s' - \rho_{bo}')\exp(-\frac{E_s'}{RT_s'}) \qquad (3)$$

(7) Radius of solid rod keeps constant and only density varies during the fuel gasification.

With these assumptions and from a reference frame at the flame front, we get the gas-phase formulation and the solid-phase formulation. The gas-phase formulation is consisted of six quasi-steady partial differential equations and a one-step overall reaction. The solid-phase formulation includes two ordinary differential equations and a Arrhenius kinetic.

From Bhattacharjee *et al.*[3], conduction through the solid is unimportant, the thermal length in the gas phase, $L_r' = \alpha_r'/V_{ref}$, is used as the length scale in both phase. Reference property such as α_r' , etc. are evaluated at a reference temperature T_r', which is the average of the adiabatic, stoichiometric flame temperature without dissociation and the ambient temperature.

The dimensionless variables and parameters are defined as:

$$L_r' = \frac{\alpha_r'}{V_{ref}} \quad x = \frac{x'}{L_r'} \quad r = \frac{r'}{L_r'} \quad u = \frac{u'}{V_{ref}} \quad v = \frac{v'}{V_{ref}} \quad \rho = \frac{\rho'}{\rho_r'} \quad p = \frac{p' - p_\infty'}{\rho_r' V_{ref}^2} \quad \mu = \frac{\mu'}{\mu_r'} \quad T = \frac{T'}{T_\infty}$$

$$E = \frac{E'}{RT_\infty} \quad \Delta H_C = \frac{\Delta H_C'}{C_p' T_\infty} \quad \mathrm{Re} = \frac{V_{ref} L_r' \rho_r'}{\mu_r'} \quad \mathrm{Pr} = \frac{\nu'}{\alpha'} \quad Le = \frac{\alpha'}{D'} \quad Da = \frac{B_F' \rho_r' L_r'}{V_{ref}}$$

$$V_f = \frac{V_f'}{V_{ref}} \quad T_s = \frac{T_s'}{T_\infty} \quad E_s = \frac{E_s'}{RT_\infty} \quad \rho_s = \frac{\rho_s'}{\rho_{s\infty}'} \quad A_s = \frac{A_s' L_r' (\rho_{s\infty}' - \rho_{bo}')}{V_{ref}\rho_{s\infty}'} \quad \dot{M}_s'' = \frac{\dot{m}_s'' L_r'}{V_{ref}\rho_{s\infty}'}$$

$$\alpha_s = \frac{\alpha_s'}{L_r' V_{ref}} \quad L_V = \frac{L_V'}{C_{ps}' T_\infty} \quad Sc = \frac{\lambda_r'}{\rho_{s\infty}' C_{ps}' L_r' V_{ref}} \quad \dot{\omega}_f = \frac{\dot{\omega}_f' L_r'}{V_{ref}\rho_r'} \qquad (4)$$

where subscript " r " represents quantities in the reference state at temperature T and " s " represents properties of the solid phase.

2.1 Governing equations

The nondimensional governing equations and boundary conditions for both phases are as following.

2.1.1 Gas-phase governing equations

All the conservation equations in the gas-phase can be written in the general form:

$$\frac{\partial}{\partial x}(\rho u \varphi)+\frac{\partial}{r\partial r}(r\rho v\varphi)=\frac{\partial}{\partial x}(\Gamma_\varphi \frac{\partial \varphi}{\partial x})+\frac{\partial}{r\partial r}(r\Gamma_\varphi \frac{\partial \varphi}{\partial r})+S_\varphi \qquad (5)$$

where φ, Γ_φ and S_φ for the different equations are shown in Table 1. In Table 1, the dimensionless reaction rate for the fuel vapor is defined as $\dot{\omega}_f = Da\rho^2 m_f m_{ox} \exp(-E/T)$.

Table 1 — Values of φ, Γ_φ and S_φ

	φ	Γ_φ	S_φ
Continuity	1	0	0
x-Momentum	u	$\frac{\mu}{\mathrm{Re}}$	$-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}(\frac{\mu}{\mathrm{Re}}\frac{\partial u}{\partial x})+\frac{\partial}{r\partial r}(\frac{\mu}{\mathrm{Re}}\frac{r\partial v}{\partial x})$
r-Momentum	v	$\frac{\mu}{\mathrm{Re}}$	$-\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}(\frac{\mu}{\mathrm{Re}}\frac{\partial u}{\partial r})+\frac{\partial}{r\partial r}(\frac{\mu}{\mathrm{Re}}\frac{r\partial v}{\partial r})-\frac{2\mu}{\mathrm{Re}}\frac{v}{r^2}$
Fuel	m_f	$\frac{\mu}{\mathrm{Re}\,\mathrm{Pr}\,Le}$	$-\dot{\omega}_f$
Oxygen	m_{ox}	$\frac{\mu}{\mathrm{Re}\,\mathrm{Pr}\,Le}$	$-s\dot{\omega}_f$
Energy	T	$\frac{\mu}{\mathrm{Re}\,\mathrm{Pr}}$	$\Delta Hc\,\dot{\omega}_f$

The six equations in Table 1 along with the equation of state $\rho T=\gamma$ are sufficient number for determining the seven unknowns u, v, p, T, ρ, m_f and m_{ox}. Viscosity μ is determined by

$$\mu=\frac{T}{\gamma} \qquad (6)$$

2.1.2. Solid-phase governing equations

Continuity equation:

$$\dot{M}_s'' = V_f \frac{d\rho_s}{dx} = A_s \frac{\rho_s-\rho_{bo}}{1-\rho_{bo}}\exp(-\frac{E_s}{T_s}) \qquad (7)$$

Energy equation:

$$\rho_s V_f \frac{dT_s}{dx}+\alpha_s\frac{d^2T_s}{dx^2} = \dot{M}_s'' L_v - \frac{2}{r_0}\mu\, Sc\frac{\partial T}{\partial r}\Big|_W \qquad (8)$$

2.2 Boundary Conditions

2.2.1 Gas-phase boundary conditions

At $x = x_{max}$:

$$u=-V_f/V_{ref} \quad v=0 \quad m_f=0 \quad m_{ox}=m_{ox\infty} \quad T=1 \qquad (9)$$

At $x = 0$:

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial x}=\frac{\partial m_f}{\partial x}=\frac{\partial m_{ox}}{\partial x}=\frac{\partial T}{\partial x}=0 \qquad (10)$$

At $r = r_{max}$:

$$u=-V_f'/V_{ref} \quad \frac{\partial\,(r\rho v)}{\partial r}=0 \quad m_f=0 \quad m_{ox}=m_{ox\infty} \quad T=1 \qquad (11)$$

At $r = r_0$:

for $0< x <x_{bo}$

$$\frac{\partial u}{\partial r}=V=\frac{\partial m_f}{\partial r}=\frac{\partial m_{ox}}{\partial r}=\frac{\partial T}{\partial r}=0 \qquad (12)$$

for $x_{bo}< x <x_{max}$

$$u=-V_f'/V_{ref} \quad v=v_W \quad T=T_s$$

$$\rho_w V_w \varphi-\frac{1}{\mathrm{Re}\,\mathrm{Pr}\,Le}\frac{\partial\varphi}{\partial r}\Big|_W=\begin{cases}\rho_w V_w & (\varphi=m_f)\\ 0 & (\varphi=m_{ox})\end{cases} \qquad (13)$$

$$\rho_w V_w = I\,\dot{M}_s'', \quad I=0.5V_{ref}\rho_{s\infty}' r_0' C_p' / \lambda_r', \qquad (14)$$

V_f', v_W and T_s are obtained from solution to the solid-phase problem

2.2.2 Solid-phase boundary conditions:

At $x = x_{max}$:

$$\rho_s=1 \quad T_s=1 \qquad (15)$$

At $x = x_{bo}$:

$$\rho_s=\rho_{bo} \quad \frac{dT_s}{dx}=0 \qquad (16)$$

where ρ_{bo} is introduced to determine the three unknowns, T_s, ρ_s and V_f. The density at burnout point is set to a specified value, ρ_{bo}= 6%. Heat flux at the interface, $(2/r_0)S_c\mu\,\partial T/\partial r\,|_W$, is obtained from the gas phase solution.

From the equations above-mentioned, we can see that gas-phase equations and solid-phase equations are coupled each other through the boundary conditions at the interface. The solution of gas-phase problem provides heat flux at the interface to the solid-phase equations and the solution to the solid-phase equations provides the boundary conditions for the gas-phase equations. At the same time, V_{ref} is not a constant during calculation. It is taken to be V_f'. However, V_f' is contained in several dimensionless parameters, such as A_s, α_s, Sc etc.. Since V_f' is not known a *priori*, an iteration scheme is needed to solve the problem in both phases. Newton Method[7] is used to solve the nonlinear ordinary differential equations in the solid phase. The gas-phase equations are solved using the SIMPLE algorithm[6]. The computer code was developed and modified by the author.

The computational domain in the gas phase was 20 × 8. For all cases, the burnout location, x_{bo}, was fixed at x = 6, and the flame tip location was allowed to change during the iteration process.

3 RESULTS AND DISCUSSION

To show the effects of diameters of fuel rods and oxygen concentration on flame spreading velocity, 9 cases are calculated. They are shown in Table 2. For all cases, properties of gas and solid are the same. All of the properties are summarized in Table 3. The gas-phase thermodynamic properties are evaluated at reference temperature T_r', i.e., $T_r' = (T_f' + T_\infty)/2$, where T_f' is the adiabatic stoichiometric flame temperature.

Table 2 Radius of fuel rod and ambient oxygen concentration for every case

case No.	r_0' (mm)	$m_{ox\infty}$	case No.	r_0' (mm)	$m_{ox\infty}$	case No.	r_0' (mm)	$m_{ox\infty}$
1	1	0.233	4	6	0.233	7	2	0.4
2	2	0.233	5	8	0.233	8	2	0.5
3	4	0.233	6	2	0.3	9	2	0.6

Table 3 Gas and solid property values

Symbol	Units	Value	Reference
ρ_r'	kg/m^3	0.227	Natl. Bur. Stan. (1955)
μ_r'	kg/m·s	5.55×10^{-5}	Natl. Bur. Stan. (1955)
λ_r'	W/m·K	9.8×10^{-2}	Natl. Bur. Stan. (1955)
C_p'	J/kg·K	1.239×10^{3}	Natl. Bur. Stan. (1955)
α_r'	m^2/sec	3.478×10^{-4}	Natl. Bur. Stan. (1955)
R	J/mole·K	8.314	Natl. Bur. Stan. (1955)
s	—	1.185	Altenkirch *et al.* (1980)
T_f'	K	2822	Altenkirch *et. al.*(1980)
T_∞'	K	298	Altenkirch *et al.* (1980)
$\Delta H_c'$	J/kg	1.674×10^{7}	Altenkirch *et al.* (1980)
E'	J/mole	7.428×10^{4}	Duh and Chen (1990)
B_F'	m^3/kg·s	3.125×10^{7}	Bhattacharjee (1990)
A_s'	1/sec	1.0×10^{10}	Frey and Tien (1977)
E_s'	J/mole	1.255×10^{5}	Frey and Tien (1977)
λ_s'	W/m·K	1.255×10^{-1}	Frey and Tien (1977)
C_{ps}'	J/kg·K	1.26×10^{3}	Altenkirch *et al.* (1980)
L_v'	J/kg	7.53×10^{5}	Altenkirch *et al.* (1980)
$\rho_{s\infty}'$	kg/m^3	750	Altenkirch *et al.* (1980)
α_s'	m^2/sec	1.328×10^{-7}	Duh and Chen (1990)

3.1 Flame Spreading Rate

Flame spreading rate (V_f) at different diameters of fuel rod is shown in Figure 2. It can be seen that the flame spreading rate decreases as the diameter of fuel rod increases.

Figure 3 shows the relationship between flame spreading rate and ambient oxygen concentration. It can be seen that flame spreading rate increases as the ambient oxygen concentration increases.

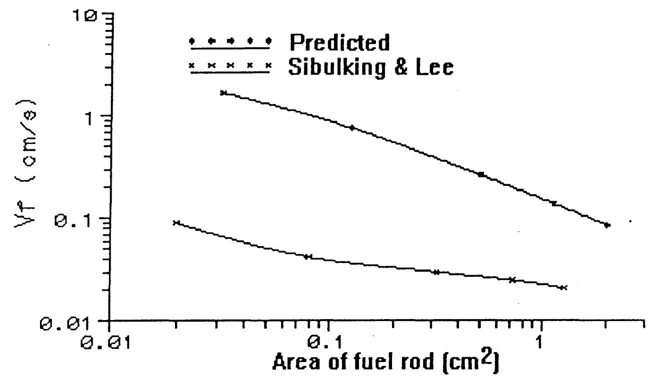

Figure 2 Flame spreading rate versus area of fuel rod

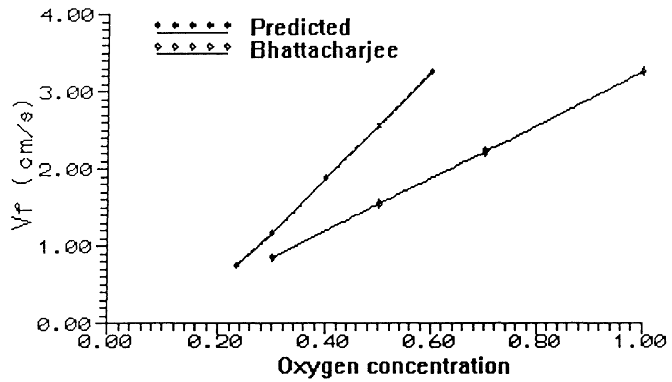

Figure 3 Flame spreading rate versus ambient oxygen concentration

3.2 Flame Structure

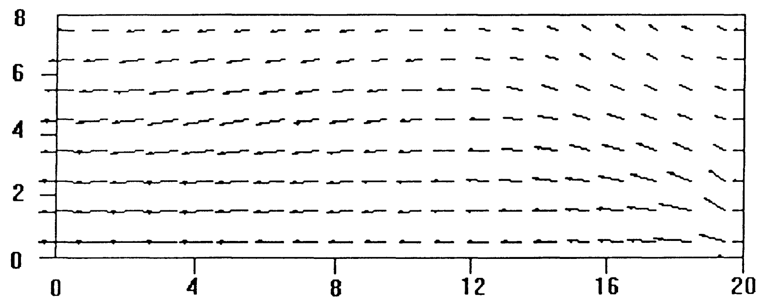

Figure 4 Velocity vectors distributions (frame attached to flame front)

Figure 4 shows the velocity vectors in gas phase of case No. 2. At reference frame attached to flame front, the flow pattern is similar to that in an opposed flow. The difference is that opposed flow velocity is the same as flame spreading rate in this case.

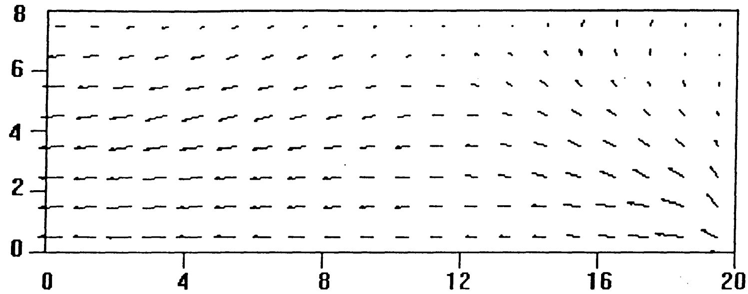

Figure 5 Velocity vectors distributions (at laboratory coordinate)

The flow pattern in gas phase at laboratory coordinate is shown in Figure 5.

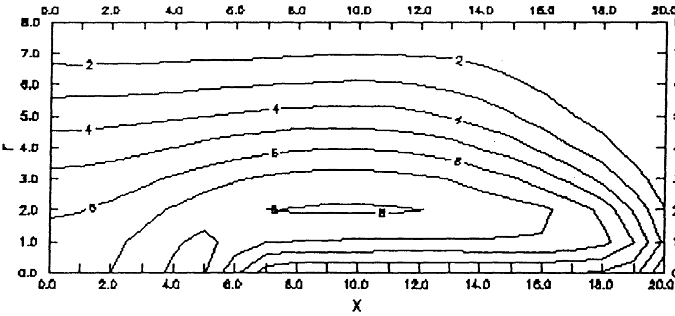

Figure 6 Isotherm distributions (dimenstionless temperature)

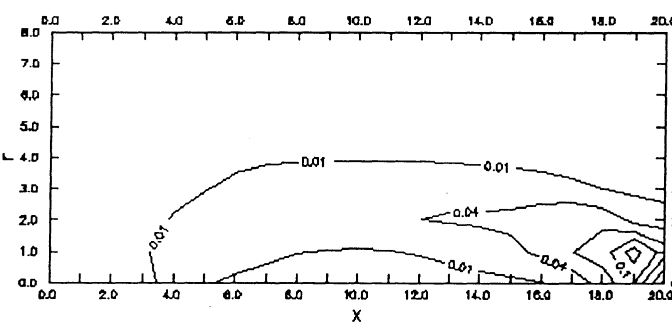

Figure 7 Dimensionless fuel reactivity rate distributions

Figure 6 shows the isotherm distributions in gas phase of case No. 2. It is found that the temperature gradient at preheat zone is great, which results a large heat flux at interface between gas phase and solid phase. (See Figure 8)

The gaseous fuel reactivity distribution, defined as $\dot{\omega}_f = Da\rho^2 m_f\ m_{ox}\exp(-E/T)$, is shown in Figure 7. It can be seen combustion takes place mainly in the pyrolysis region. Since the fuel vapors are not completely consumed in the pyrolysis region, the flame extends into the downstream wake region.

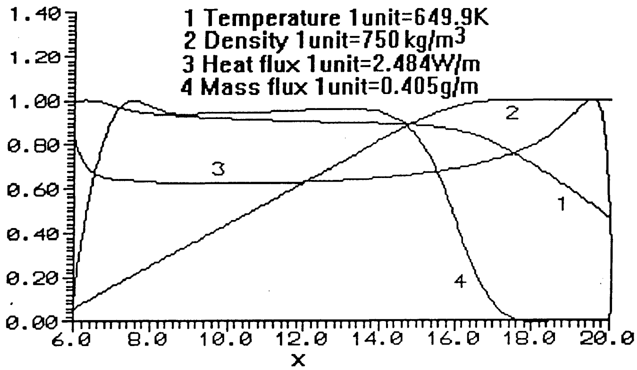

Figure 8 Solid phase response

The response of solid fuel is shown in Figure 8. It includes the distributions of fuel temperature, density, heat flux and the mass flux from the gas phase. It can be seen that the temperature of fuel rod increases rapidly as it reaches the flame front, and then increases gradually. The density of fuel rod keeps constant first, then it decreases almost linearly.

An interesting phenomenon is that the maximum q_{in} takes place in preheat region. Since the gaseous fuel reactivity, $\dot{\omega}_f$, is large, the temperature difference between gas phase and solid phase get its maximum, which results the maximum of q_{in}.

3.3 Comparisons with Experiment and Other Related Studies

The most relevant experimental study for comparison to this work is that of Sibulkin and Lee[4]. In their paper, flame spreading velocity measurements were made using PMMA cylinders as fuel rods. The comparison between predicted results and that of Sibulkin and Lee[4] is shown in Figure 2. It can be seen that predicted results are the same as that of Sibulkin and Lee qualitatively. The difference between them is that predicted results are based on filter paper.

The most relevant numerical study is that of Bhattacharjee *et al*[3]. In their paper, flame spreading over a thermally-thin fuel bed is calculated. Comparisons between their results and ours are shown in Figure 3. It can be seen that relationship between flame spreading rate and ambient oxygen concentration given by Bhattacharjee *et al* is almost the same as that in this paper. The difference between them is that flame spreads faster over rod fuel than over fuel bed at same ambient oxygen concentration.

4 CONCLUSIONS

A mathematical model for flame spread over a cylinder fuel in a quiescent environment is developed and solved numerically to predict the structure and spreading rate of flame. In a reference frame attached to the flame, a quasi-steady state in the gas phase with respect to the solid phase can be established and the solid-phase equations become steady and the fuel is fed into the flame with velocity V_f. The gas-phase equations are coupled to the solid-phases ones through the spreading rate, which appears as an eigenvalue in both phases, and the interfacial boundary conditions. The mathematical system consists of the two-dimensional momentum, energy, species and continuity equations in the gas phase. A one-step overall chemical reaction with second-order Arrhenius kinetic is also included. In the solid phase, the energy balance equation and mass balance equation are included. The solid-phase mass balance includes an Arrhenius expression for fuel pyrolysis.

The mathematical model is solved numerically. SIMPLE[6] algorithm is adopted to solve the partial differential equations in the gas phase and Newton Method[7] with an under-relaxation is used to solve the nonlinear ordinary differential equations in the solid phase. Iteration scheme is made between gas phase and solid phase.

According to the simulation results, the flame spreading rate decreases as the diameter of fuel rod is increased. The flame spreading rate decreases as the ambient oxygen concentration decreases.

ACKNOWLEDGMENTS

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