

## ON THE PREDICTION OF FLAME SPREAD RATE OVER THE COMBUSTIBLE MATERIALS

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### ABSTRACT

The propagation of two-dimensional diffusive flame over combustible material is studied numerically by solving appropriate conservation equations for the reacting flow coupled with the energy equation in the solid fuel. Two kinds of mathematical descriptions are considered. Firstly, boundary layer approach is taken into account but in some cases (specifically, when free-stream velocity is not sufficient to damp the molecular heat and mass transfer in the upwind direction), the flame spread features can not be described correctly by this approximation. Hence, full two-dimensional elliptic equations are solved. Among other aspects of flame spread process, presented study has been focused mainly on the central issue of theoretical analysis of considered phenomena that consists in the prediction of steady flame spread rate. Because the solving of unsteady elliptic equations is fairly difficult from the view-point of computational efforts, mathematical model is formulated in the steady-state coordinate system fixed on the flame front. Here, set of steady equations is taken into account where steady flame spread rate appears as an eigenvalue. Generally accepted method of eigenvalue problem's solution relates to the using of integral mass balance in the solid fuel. As it was shown, two-dimensional eigenvalue problem has not unique solution if regression of burning surface is not neglected. It means that fuel's mass balance is kept for any assigned value of flame spread rate. A new algorithm for the prediction of steady flame spread rate is proposed by using the non-equilibrium thermodynamic approach.

The flame spread over PMMA slabs and thin samples of paper is studied. The influence of some parameters such as free-stream velocity and ambient mass fraction of oxygen on the flame spread rate is investigated.

### INTRODUCTION

The problem of theoretical investigation of flame spread over solid fuels was widely studied by different authors [1-5]. Among lot of physical aspects surrounding this phenomena, present study has been mainly concentrated on the theoretical prediction of the steady flame spread rate. As it was noticed [3], quite difficult two-dimensional eigenvalue problem has been arisen if steady-state coordinate system fixed on flame front is considered. Alternative approach is based on the unsteady equations in the

coordinate system fixed on solid fuel. Using this approach [4,5] one can avoid eigenvalue problem but significant difficulties are arisen to solve two-dimensional unsteady equations.

At presented paper only steady equations in the coordinate system fixed on flame front are taken into account and eigenvalue problem is studied especially. In the originating work [2] the solution of eigenvalue problem was received by using the integral mass balance of solid fuel. In this paper the ratio of surface regression to the flame spread rate was assumed negligible that allowed to reduce full two-dimensional eigenvalue problem to one-dimensional. In this regard, present study has been intended to develop an algorithm for the prediction of flame spread rate basing on the analysis of full two-dimensional eigenvalue problem where flame spread rate and solid fuel's pyrolysis rate relate to each other as a two eigenvalues.

#### FLAME SPREAD MODEL

The configuration of considered flame spread model is shown in Fig.1. The horizontal flame spread is shown here but presented analysis can be applied also to downward and even upward flame spread since the existence of steady propagation is the single condition for suitability. The general form of equations is:

gas-phase

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = A \frac{\partial \lambda}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial \lambda}{\partial y} \frac{\partial T}{\partial y} + \frac{Q}{C} \rho W_g \quad (1)$$

$$\rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = A \frac{\partial Le \lambda}{\partial x} \frac{\partial Y_i}{\partial x} + \frac{\partial Le \lambda}{\partial y} \frac{\partial Y_i}{\partial y} - \nu_i \rho W_g \quad (2)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = A \frac{\partial \mu}{\partial x} \frac{\partial u}{\partial x} + \frac{\partial \mu}{\partial y} \frac{\partial u}{\partial y} - \frac{\partial p}{\partial x} \quad (3)$$

$$A \rho u \frac{\partial v}{\partial x} + A \rho v \frac{\partial v}{\partial y} = A \frac{\partial \mu}{\partial x} \frac{\partial v}{\partial x} + A \frac{\partial \mu}{\partial y} \frac{\partial v}{\partial y} - \frac{\partial p}{\partial y} \quad (4)$$

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (5)$$

$$p = \rho R T \quad (6)$$

solid fuel

$$u_f \frac{\partial T}{\partial x} + v_s \frac{\partial T}{\partial y} = \frac{\lambda}{\rho C} \left[ A \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \frac{Q_s}{C} \rho W_s \quad (7)$$

boundary conditions

$$y=0: u=u_f, \quad \rho v = \rho_s v_s, \quad T_g = T_s \quad (8)$$

$$-\lambda_g \frac{\partial T}{\partial y} + (\rho v C T)_g = -\lambda_s \frac{\partial T}{\partial y} + (\rho v C T)_s \quad (9)$$

$$-\frac{Le}{C} \lambda \frac{\partial Y_i}{\partial y} + \rho v Y_i = \rho_s v_s Y_{s,i} \quad (10)$$

Here  $T$  is temperature,  $u, v$  are velocity components,  $Y_i$  is mass fractions where  $i=(O, F)$ ,  $O$  and  $F$  denote oxygen and gasified fuel respectively,  $\rho$  is density,  $\lambda$  is thermal conductivity,  $\mu$  is viscosity,  $C$  is specific heat,  $p$  is pressure,  $R$  is specific gas constant,  $Q$  is effective heat of reaction,  $Le$  is Lewis number,  $u_f$  and  $v_s$  are steady flame spread rate and linear pyrolysis rate respectively,  $A=0$  for boundary layer approach and  $A=1$  for full two-dimensional elliptic equations.

The gas-phase combustion and solid fuel pyrolysis both are described by Arrhenius type formula:

$$W_g = Y_F Y_O k_g \exp(-E_g / R_0 T_g) \quad (11)$$

$$W_s = k_s \exp(-E_s / R_0 T_s) \quad (12)$$

#### EIGENVALUE PROBLEM

Assuming pyrolysis product is gasified from the burning surface as soon as it is generated, the linear rate of pyrolysis is determined from the fuel's mass conservation in the form [3]:

$$v_s = \int_{-L}^0 W_s dy \quad (13)$$

The value of pyrolysis rate  $v_s$  in Eq.(13) relates to some cross-section of fuel sample in the  $x$ -coordinate. The mass conservation equation describing the whole fuel sample is (see Fig.1):

$$u_f L = \int_0^{x_m} v_s(x) dx + u_f (L - \delta_m) \quad (14)$$

which yields

$$u_f \delta_m = \int_0^{x_m} v_s(x) dx \quad (15)$$

Here  $x_m$  is burnout point on the fuel surface beyond which pyrolysis no occurs.

The thickness of virgin material decreases along the  $x$ -coordinate

$$L(x) = L(0) - \delta(x) \quad (16)$$

and thickness of gasified material is defined as

$$\delta(x) = \int_0^t v_s(x) dt \quad (17)$$

On the other hand, steady-state coordinate system is fixed on the flame front and determination of steady flame spread rate yields

$$dx = u_f dt \quad (18)$$

Replacing  $dt$  from Eq.(18) to Eq.(17) we receive

$$\delta(x) = \frac{1}{u_f} \int_0^x v_s(x) dx \quad (19)$$

and for the whole sample

$$\delta_m = \frac{1}{u_f} \int_0^{x_m} v_s(x) dx \quad (20)$$

Finally, replacing  $\delta_m$  from Eq.(20) to Eq.(15) we get an identity that means that fuel's mass conservation is kept for any assigned value of flame spread rate  $u_f$  and eigenvalue problem has not unique solution relatively to the spread rate. Therefore, some additional correlations are needed.

#### BOUNDARY LAYER APPROACH

Considering the eigenvalue problem presented in the previous section, the following algorithm is proposed for the prediction of flame spread rate. The investigation [6] of dependence  $v_s(u_f)$  has shown that there is an extremum at which the pyrolysis rate  $v_s$  reaches its maximum corresponding to some value of flame spread rate  $u_f$ . The last one is considered as a sought steady value. Because boundary layer equations are taken into account and terms describing heat transfer in the upwind direction are neglected, Eqs.(1)-(4) can not be integrated from the point  $x=0$  in Fig.1 [7]. So, to begin the calculations some profiles of predicted variables must be assigned at some cross-section  $x_0$  in Fig.1. Actually it means that in the case of opposed flow flame propagation the flame spread rate can not be predicted by theoretical study purely because boundary layer equations do not

include any terms that describe mechanism of heat transfer in the flame spread direction. Therefore, only an influence of some parameters on the flame spread rate can be studied by boundary layer approach while one point of this dependence in any case must be guessed by choosing the initial profiles that satisfy experimental result.

Some results of the investigation of flame spread over PMMA slabs are presented in Fig.2-3. The data of Fig.2 are quite satisfactory while Fig.3 has not shown so good agreement. Author does not hesitate to show achieved results even if its have a manner such as presented in Fig.3 because the limit of proposed method for flame spread rate prediction must be settled clearly.

#### FULL TWO-DIMENSIONAL EQUATIONS

Here, a mathematical model that includes full two-dimensional equations are investigated. The study of influence of free-stream velocity on the flame spread rate by using the method described in previous section (searching the extremum of  $u_g(u_f)$  distribution) was unsuccessful as it was shown in Fig.3 for boundary layer approach. Then, a new approach for the prediction of flame spread rate was developed by using the non-equilibrium thermodynamic analysis [8]. By this approach the steady flame propagation is identified with the stationary state of irreversible thermodynamic system that is characterized, according to theorem of Prigogine, by minimal entropy production [9]. The result achieved by this approach is shown in Fig.4.

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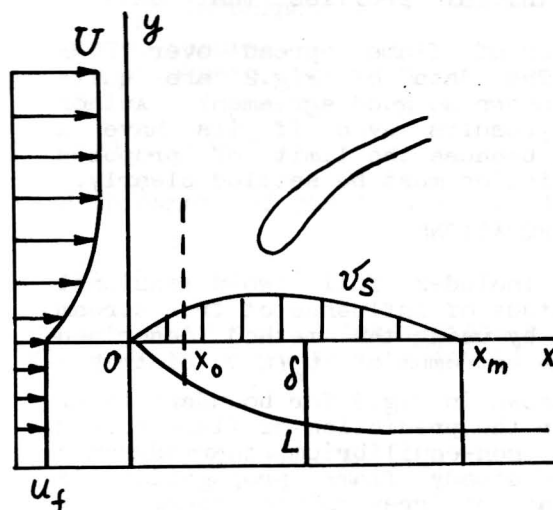


Fig.1 Flame spread model

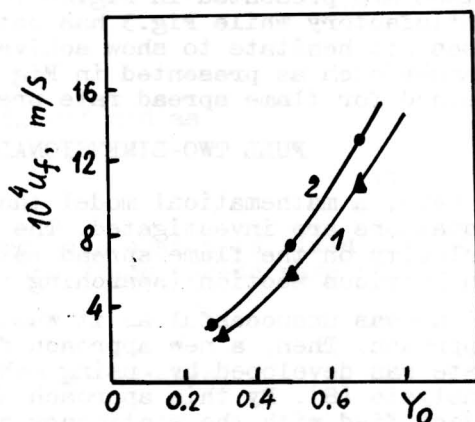


Fig.2 The influence of ambient oxygen mass fraction on the flame spread rate.  
1 - present calculation,  
2 - empiric formula [10].

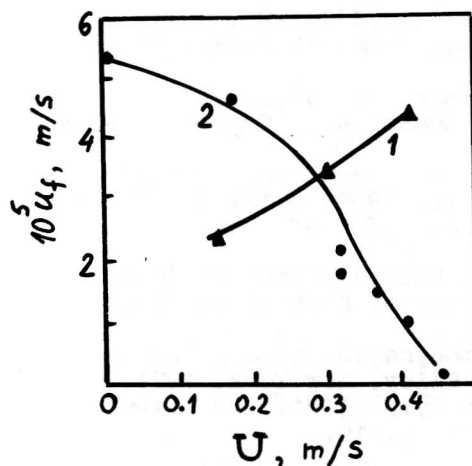


Fig.3 The Influence of free-stream velocity on the flame spread rate.  
1 - present calculation  
2 - measurements [6]

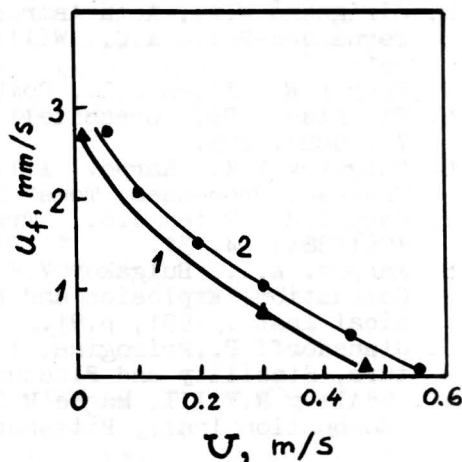


Fig.4 The influence of free-stream velocity on the flame spread rate.  
1 - present calculation  
2 - calculation [4]