PREDICTION OF THERMAL RESPONSES OF REINFORCED CONCRETE BEEMS EXPOSED TO FIRE BY USING F.E.M.

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ABSTRACT

The thermal response of reinforceed concrete beams exposed to fire are analytically studied by using finite element method (FEM) in this paper. Through tests of concrete thermal parameters, calculating models of thermal paramters are established On the basis of calculating models, a method which combines the finite element with the finite difference is used to solve heat conduction equation and a computer program used for calculating the thermal responses is presented and verified against test data.

INTRODUCTION

When a reinforced concrete beam is exposed to fire, fire temperature will invade the boundary of the reinforced concrete beam, then develop within beam. The thermal responses of reinforced concrete beam have relied on the results of standard fire tests for establishing fire endurance ratings[1]. In recent years, various methods for predicting thermal responses to fire analytically have been developed[2–3]. The methods by which the finite defference is used vary in the extent of simplifieation and approximation introduced into the analysis. In this paper, a method which combines the finite element with the finite difference are proposed to analytically calculate thermal responses of reinforced concrete beams exposed to fire.

Fire elevated temperature curve express the time during fire and fire temperature at room relationship curve. Many scholars research on simulating fire through a lot of actural fire test and defined various time–temperature relationship curves. For example, ACI Committee 216 proposed ASTM E119 curve. The test time–temperature relationship curves are shown in Fig.1 in this papre.

HEAT CONDUCTION EQUATION AND THERMAL PROPERTIES

Thermal distribution within concrete beam section is assumed to be governed by the three–dimensional heat conduction equation:

$$\lambda(T)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c\frac{\partial T}{\partial t} \quad (1)$$

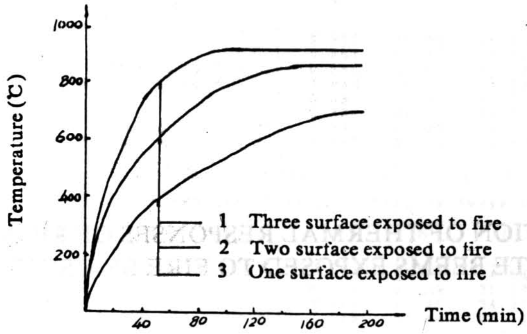

Fig.1 Test time–temperature relationship curves

Where x, y and z are the point coordinate in a three–dimensional space, T is the temperature, and t is the time. This equation is derived from elementary thermodynamic principals. The thermal properties of a material are characterized by three parameters: λ–conduetivity, c–heat capacity and ρ–density. The paramenters is referred to as thermal property parameters. Through test of thermal property for concrete, the parameter of conductivity λ is given by the following:

$$\lambda(T) = 1.6 - 0.706 \times 10^{-5} T \qquad (W / m℃) \tag{2}$$

and the parameters of heat capacity c and density ρ are regarded as constant in calculating.

THERMAL RESPONSE ANALYSIS BY USING FINITE ELEMENT AND DIFFERENT METHOD

In accordance with simulating actural fire, equation(1) can be simplified two–dimensional heat conduction equation:

$$\lambda(T)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \rho c \frac{\partial T}{\partial t} \tag{3}$$

This equation is called as Poisson equation. The field of temperature within concrete beam section is changeable, because the boundary conditions vary with time (temperature).

1. The Boundary and Initial Conditions

In order to get a unique solution of two–dimensional heat conduction, it is necessary to provide the boundary and initia! condition which is called as determinate solution conditions.

The first boundary condition means that the temperature around boundary of object known as follows(Fig.2)

$$T|_{\Gamma} = T_w$$
$$T|_{\Gamma} = f(x,\ y,\ t) \tag{4}$$

where $T_w$is the known temperature (constant), f(x,y,t) is known temperature function varied with time.

The second boundary condition means that the heat flux (heat flow per unit area of exposed surface) around boundary of object is known as follows:

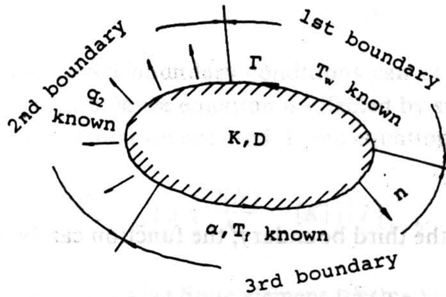

Fig.2 Boundary Conditios

$$-\lambda \frac{\partial T}{\partial n}\bigg|_{\Gamma} = q_2$$
$$-\lambda \frac{\partial T}{\partial n}\bigg|_{\Gamma} = q_2(x,\ y,\ t) \tag{5}$$

Where q_2 is the known heat flux (constant), $q_2(x,y,t)$ is the known heat flux function varied with time.

The third boundary condition means that the flow mass temperature and the coefficient of heat transter is know as follows:

$$-\lambda \frac{\partial T}{\partial n}\bigg|_{\Gamma} = \alpha(T - T_f)\big|_{\Gamma} \tag{6}$$

The flow mass temperature T_f and the coefficient of heat transfer α can be constant or function. Initial condition is shown as the following:

$$T|_{t=0} = T_0$$
$$T|_{t=0} = \psi(x,\ y) \tag{7}$$

Where T_0 expresses that initial temperature of object is mean, $\psi(x,y)$ expresses that the temperature is nonmean.

2. Steady and Nonsteady Temperature Field

For steady temperature field, by using variational principle, we solve function

$$J[T(x,y)] = \iint_D \frac{\lambda}{2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] dxdy \tag{8}$$

the limit value of curved surface $T(x,y)$, where temperature curved surface around boundary Γ of district D is known as follows:

$$T(x,y)|_{\Gamma} = f(x,y) \tag{9}$$

Through Euler's equation variation, we get conclusion that the limit value function $T(x,y)$ given by variation of the function (8) at boundary condition(9) is the solution under Laplacian condition as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{10}$$

$$T(x,y)|_\Gamma = 0$$

For non steady temperature at the third boundary; the function can be shown as follows:

$$J[T(x,y)] = \iint_D \frac{\lambda}{2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] dxdy + \oint_\Gamma \alpha\left(\frac{1}{2}T^2 - T_f T\right) ds \tag{11}$$

it's solution:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad -\lambda\frac{\partial T}{\partial n}|_\Gamma = \alpha(T - T_f) \tag{12}$$

For the third boundary of nonsteady temperature field, the differential equations are given as follows:

$$\begin{cases} \dfrac{\partial T}{\partial t} = \dfrac{\lambda(T)}{\rho c}\left(\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2}\right) \\ t = 0 \cdot T = f(x,y) \quad (known) \\ -\lambda(T)\dfrac{\partial T}{\partial n}|_\Gamma = \alpha(T - T_f) \end{cases} \tag{13}$$

It is called as parabolic equation which is difficultly solved at present. An approach is adopted which make t temporary stationary to consider functional variation (under some time condition) where $\partial T / \partial t$ is noly a function of place, then consider t various to develop $\partial T / \partial t$ by using difference equations, The function with respect to equation(13) is:

$$J[T(x,y,t)] = \iint_D \left\{\frac{\lambda(T)}{2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] dxdy + \rho c\frac{\partial T}{\partial t}T\right\} dxdy + \oint_\Gamma \alpha\left(\frac{1}{2}T^2 - T_f T\right) ds \tag{14}$$

3. Finite Element and Difference Method

For nonstead temperature field, elements function's dierction with respect to temperature is given as follows:

$$\left\{\frac{\partial T}{\partial t}\right\}^e = [K]^e\{T\}^e + [N]\left\{\frac{\partial T}{\partial t}\right\}^e - \{P(t)\}^e \tag{15}$$

In accordance with the condition of function given limit value, the matrix equation is derived as following:

$$[K]\{T\} + [N]\left\{\frac{\partial T}{\partial t}\right\} = \{P(t)\} \tag{16}$$

The heat flow equation and associated boundary conditions can be solved by the finite element method. Solution of the matrix heat balance equation is effected by step–by–step time integration where Crank–Nicalson's difference equation are used. From equation(16), we have:

$$\left([K]+\frac{2[N]}{\Delta t}\right)\{T\}_t=(\{P\}_t+\{P\}_{t-\Delta t})+\left(\frac{2[N]}{\Delta t}-[K]\right)\{T\}_{t-\Delta t} \tag{17}$$

The property of the solution method is that finite element mesh is used in space region and finite difference mesh is used in time region. A computer program that implements the above method has been developed and is used to calculate the thermal responses (refer to Fig.4).

COMPARISON OF CALCULATED RESULTS WITH TEST RESULTS

1. Test Method

Through fire tests of concrete beam, temperature distribution within concrete beam section is measured. The test elevated temperature curves are shown in Fig.1 including one surface fire exposure, two surface fire exposure and three surface fire exposure curves. The boundary conidions are shown in Fig3. The size of concrete beam section is 150 × 200mm.

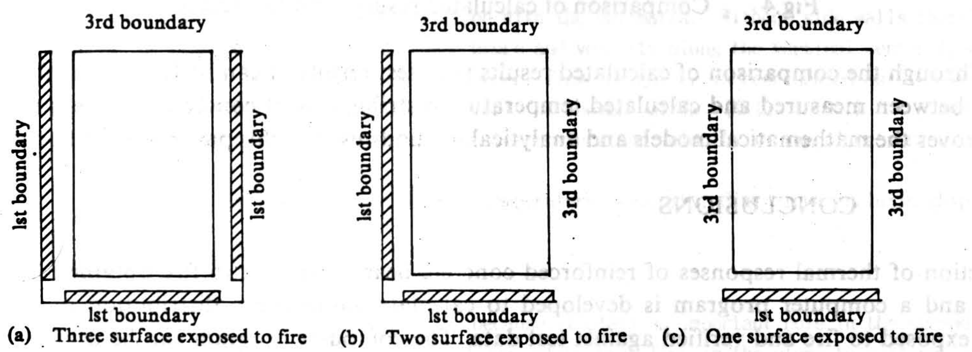

Fig. 3 Test boundary condition

Temperatures within concrete beam section is measured by using thermocouples (refer to Fig4). Measured results are shown in Fig.4.

2.Theoretical Calculations

In accordance with the computer program which input concrete thermal conductivity (refer to equation(2)), heat capacity 0.025 Kcal / N^{0}C, density 24KN / m^{3}and the boundary and initial conditions, temperature distributions of the concrete beam section are calculated (refer to Fig.4).

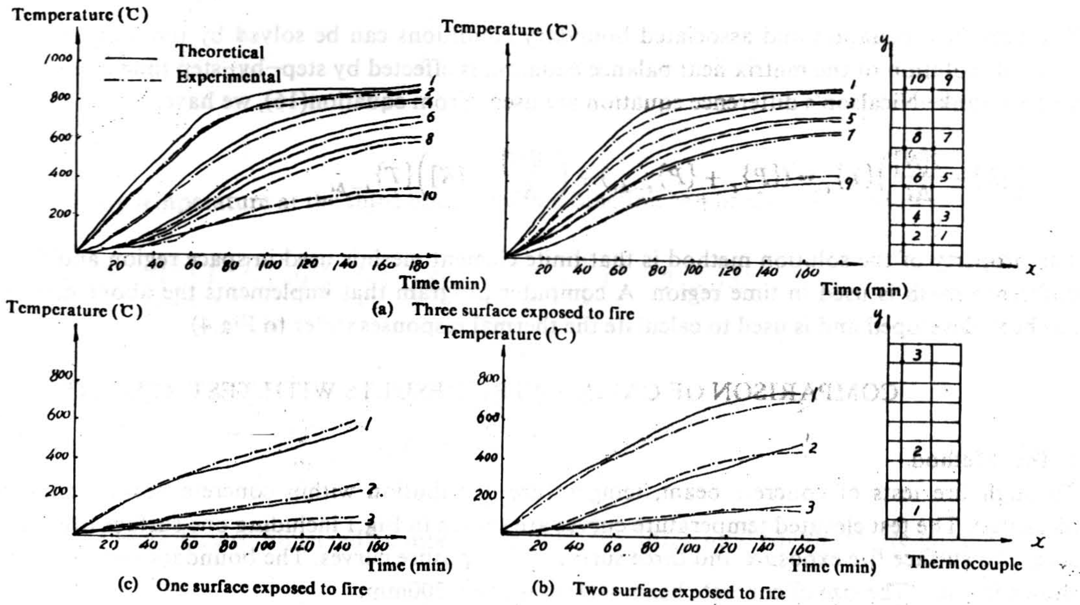

Fig.4 Comparison of calculated results with test results

Through the comparison of calculated results with test results, it can be found that the correlation between measured and calculated temperature distributions of concrete beam is very close and proves the mathematical models and analytical methods used in this paper to be rational.

CONCLUSIONS

Prediction of thermal responses of reinforced concrete beams exposed to fire obtained by using FEM and a computer program is developed to calculate temperature distribution of concrete beam exposed to fire and verified against test data. The computer program can be used to calculate thermal responses in different elevated temperature curves, different boundary conditions and different kind of concrete structures.

REFERENCES

1. Lie,T.T., Fire and Building, Division of Building Research, National Research Cuncil of Canada, (1972)
2. Bresler, B. and Iding, M.S., Fire Response of Prestressed Concrete Members, Fire Safety of Concrete structures ACI sp–80, (1983)
3. Allen, D.D. and Thorne, C.P., The thermal Conductivity of Concrete, Magazine of Concrete Research, Vo.14, No.43, Mar., (1983)