ANALYSIS OF SEISMIC FIRE

Yoshiro Namba, Toyohiro Omori and Kenjiro Yasuno
(Department of Architecture, Faculty of Engineering, Kinki University
1-ban, Takayaumenobe, Higashihiroshima-City, Hiroshima, 729-17, Japan)

ABSTRACT

This paper deals with the number of the seismic fire. The usual least squares method of curve fitting is not suitable to the analysis for counting numbers. Then more suitable method, which is based on the maximum likelihood estimation, is introduced for the curve fitting problem concerning to the non-negative discrete data such as the number of the seismic fire. It is concluded that this method is better than the usual method and that the results support the assumption of the new method. And the methodology which can give the most appropriate formula in view of statistical model is referred.
Keywords ; Seismic fire, earthquake, curve fitting, least squares method, maximum likelihood, estimation, statistical model, data analysis

1. INTRODUCTION

Several formulas on the number of the seismic fire have been reported till now[1-7]. However, it can be pointed out that former studies have a few of imperfections theoretically and it is insufficient to discuss for propriety of the formulas statistically. Then we consider more theoretically and deal with the curve fitting problem concerning to the non-negative discrete data as synthetically as possible. And the purpose of this paper is to investigate the methodology which can give the most appropriate formula in view of statistical model. Although we deal with the seismic fire, we confine to the number of it. It is also a significant problem to discuss the mechanism of the seismic fire, but it is omitted owing to limited space.

2. REVIEW OF LITERATURE

H.Kawasumi[1] researched the relationship between the ratio of serious fires outbreak and the ratio of destroyed houses following Kanto earthquake(1923) in japan. So-called Kawasumi formula is expressed as below after M.Kobayashi[2].

$$\log Y = 0.773 \log X - 1.854 \quad (1)$$

where X is the ratio of destroyed houses and Y is the ratio of serious fires outbreak. Here serious fires means fires which are not extinguished immediately, and which spread to surrounding buildings. Next, H.Mizuno[3,4] has reserched the following relationship based on the data from 13 different earthquakes dating from 1923 to 1974 in Japan.

$$\log \{y/(\alpha \cdot N)\} = a \cdot \log(X) + r \cdot s + b \quad (2)$$

where y is the number of seismic fire, N is the number of all houses, X is the ratio of destroyed houses, α is a factor of using fire instruments and s is a factor of season. a, b, and r are coefficients. The values of coefficients and factors are omitted.
Eq.(2) can be changed as follows:

$$y = \alpha \cdot \beta \cdot B \cdot N \cdot X^{a} \quad (3)$$

where $\beta = e^{b}$ and $B = e^{rs}$. Furthermore

$$y/N = c \cdot X^{a} \quad (4)$$

or

$$\log Y = a \cdot \log X + \log c \quad (5)$$

where $c = \alpha \cdot \beta \cdot B$ and $Y = y/N$. One of eq.(5) is as follows:

$$\log Y = 0.429 \log X - 6.231 \quad (6)$$

H.Kawasumi and H.Mizuno used the method of least squares but Building Research Institute[5], Ministry of Construction, Japan, proposed the method of safety side estimation, is based on the method of least squares, and analyzed the following relationship based on the same data of H.Mizuno.

$$\log Y = 0.299 \log X - 4.656 \quad (7)$$

One the other hand, M.Kobayashi[2] has pointed out that other relationships have higher correlation coefficients than those of the above type functions.

$$Y = 0.00348X - 0.0163 \quad (8)$$

$$Y = 0.0114 \log X - 0.0210 \quad (9)$$

where X is the percentage of destroyed houses and Y is the percentage of serious fires outbreak.
Lastly, O.Koide[6] used the maximum likelihood estimation with the data supposed to be the Poisson distribution and found following formula.

$$Y = 0.020X - 0.872 \quad (10)$$

where X is the number of destroyed houses per 10000 houses and Y is the number of serious fires outbreak per 10000 houses. And it is reported by O.Koide that the data of seismic fire in the vicinity to zero follow the Poisson distribution. Here, we can summarize following two points of the problem in former studies on the number of seismic fire:

a. Is the analysis method suitable theoretically?
b. What type of function of formula is appropriate?

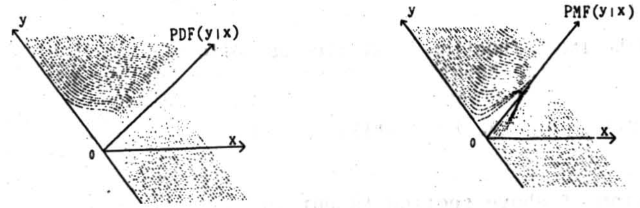
3. THEORETICAL REMARKS

(1) Maximum Likelihood Estimation[8]

We suppose now that X is a random variables, discrete or continuous, whose distribution depends upon the parameters $\theta_1, \theta_2, \cdots \theta_p$. Let $x_1, x_2, \cdots x_n$ be an observed sample. The idea of maximum likelihood estimation of the parameters $\theta_1, \theta_2, \cdots \theta_p$, which characterize a random variable X, is to choose the values which make the observed sample $x_1, x_2, \cdots x_n$ most probable. If X is discrete, the probability that a random sample consists of these values exactly is given by

$$L = \prod_{i=1}^{n} f(x_i \mid \theta_1, \theta_2, \cdots \theta_p) \quad (11)$$

where $f(\cdot)$ is the probability mass function of X. L is called the likelihood function and is a function of the parameters $\theta_1, \theta_2, \cdots \theta_p$. If X is continuous, $f(\cdot)$ is the probability density function of X. In many cases it is more convenient to work with $l = \ln L$. If we suppose the random variable X has a normal distribution, the maximum likelihood estimation is the same as the method of least squares. The normal distribution consists of the continuous data from $-\infty$ to $+\infty$ theoretically. By the way, we have often applied the method of least squares to the curve fitting problem till now even if the data are discrete. It is because that the distribution can be approximated by normal one under certain conditions. But if the data are confined to the non-negative integer and in the vicinity to zero, it is known by Fig.1 that the distribution is not suitable by reason of being out of range. T.Awaya[9-11] have developed the new method which is suitable to analysis for the intensity estimation of such data with low statistics. This method can be precisely applied to the data which follow the Poisson distribution and is based on the maximum likelihood estimation. He has analyzed the number of radioactive nuclear and of their half-lives. It has then been concluded that the method gives good results. The method of least squares and Awaya method are summarized as follows.



(a) Normal distribution (b) Poisson distribution

Fig.1 The typical example of distribution function

(2) The method of least squares[12,13]

let $(x_1,y_1),(x_2,y_2),\cdots,(x_n, y_n)$ be the data series. The fitting function is written by

$$y=g(x \mid a_1,a_2,\cdots a_m) \tag{12}$$

where $a_1,a_2,\cdots a_m$ are the parameters to be searched. Then the likelihood function is given by

$$L=\prod_{i=1}^{n}\frac{1}{\sqrt{2\pi}\sigma}\exp\left\{-\frac{1}{2\sigma^2}[y_i-g(x_i \mid a_1,\cdots a_m)]^2\right\} \tag{13}$$

$$=\left\{\frac{1}{\sqrt{2\pi}\sigma}\right\}^n\exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}[y_i-g(x_i \mid a_1,\cdots a_m)]^2\right\} \tag{14}$$

where σ^2 is the variance. The logarithmic likelihood function can be written as follows:

$$l=-\frac{n}{2}ln(2\pi)-\frac{n}{2}ln(\sigma^2)-\frac{1}{2\sigma^2}\sum_{i=1}^{n}[y_i-g(x_i \mid a_1,\cdots a_m)]^2 \tag{15}$$

The maximum likelihood estimators of $a_1,\cdots a_m$ are the values that maximize l. The optimal values of the parameters can be obtained by solving normal equations, i.e. a simultaneous equation in m unknowns which is obtained by setting the partial derivatives of S with respect to the parameters $a_1,\cdots a_m$ equal to zero. If g is a linear function with respect to the parameters, it is easy to solve it. If g is a nonlinear function with respect to the parameters, it is difficult to solve it. Then we must substitute the initial approximate values of the parameters and repeat the try-and-error calculation for that case. It is important to take care in choosing these initial values in order to guarantee fast convergence and to get reliable results. If we could solve it anyway, we describe the estimators as $\hat{a}_1,\cdots \hat{a}_m$. On the other hand, in order to maximize l the following equation is essential.

$$\left.\frac{\partial l}{\partial \sigma^2}\right|_{\sigma^2=\hat{\sigma}^2}=-\frac{n}{2\hat{\sigma}^2}+\frac{1}{2(\hat{\sigma}^2)^2}\sum_{i=1}^{n}[y_i-g(x_i \mid a_1,\cdots a_m)]^2=0 \tag{16}$$

Then

$$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}[y_i-g(x_i \mid \hat{a}_1,\cdots \hat{a}_m)]^2 \tag{17}$$

(3) Awaya method

(A) the case of the data with the Poisson errors in ordinate

We use the same data and function as mentioned previous section. The datum y_i (i=1, …

n) is supposed to be a sample from the Poisson distribution. Then the likelihood function is

$$L=\prod_{i=1}^{n}\frac{\{g(x_i \mid a_1\cdots a_m)\}^{y_i}}{y_i\,!}\exp\{-g(x_i \mid a_1\cdots a_m)\} \tag{18}$$

and the logarithmic likelihood function is written as follows:

$$l=\sum_{i=1}^{n}[y_i\,ln\{g(x_i \mid a_1\cdots a_m)\}-ln(y_i\,!)-g(x_i \mid a_1\cdots a_m)] \tag{19}$$

The similar calculation of above section is put into operation and the estimators of parameters $\hat{a}_1\cdots\hat{a}_m$ are solved.

(B) the case of the data with the Poisson errors in ordinate and abscissa; called two-dimensional fitting

This part of section is improved the Awaya method by authors. Let the pairs of observed data be $(x_1, y_1), \cdots, (x_n, y_n)$ and the datum y_i $(i=1\cdots n)$ and the datum x_i $(i=1\cdots n)$ are supposed to be observed variables from the Poisson distributions whose parent means are Φ_i and Ψ_i, respectively. Then the likelihood function is

$$L=\prod_{i=1}^{n}\frac{\Psi_i^{x_i}}{x_i\,!}\exp(-\Psi_i)\frac{\Phi_i^{x_i}}{y_i\,!}\exp(-\Phi_i) \tag{20}$$

and the logarithmic likelihood function is written as follows:

$$l=\sum_{i=1}^{n}[x_i\,ln(\Psi_i)-ln(x_i\,!)-\Psi_i+y_i\,ln(\Phi_i)-ln(y_i\,!)-\Phi_i] \tag{21}$$

Since both x_i and y_i include errors, the function to be fitted to the data should be written as an implicit function of two variables, x and y.

$$h(x,y \mid a_1,\cdots a_m)=0 \quad \text{or} \quad y-g(x \mid a_1,\cdots a_m)=0 \tag{22}$$

In order to get the maximum values of l, it is convenient to use the method of Lagrange multipliers. Then the logarithmic likelihood function is changedas follows:

$$l=\sum_{i=1}^{n}[x_i\,ln(\Psi_i)-\Psi_i+y_i\,ln(\Phi_i)-\Phi_i+\lambda_i\,h(x_i, y_i \mid a_1,\cdots a_m)] \tag{23}$$

where λ_i is Lagrange's multipliers and the term which play no part in calculating are omitted. The procedure to get the best values of parameters is the same as the previous section. If the estimators of parameters $\hat{a}_1, \cdots \hat{a}_m$ are obtained, the maximum value of the logarithmic likelihood function can be given.

(4) Indexes of appropriateness and propriety of fitting function

We consider statistically the following indexes of appropriateness and propriety of fitting function.

a. Sum of squares residual; S
b. Coefficient of correlation between datum y_i and estimator $\hat{y}_i$; R
c. The maximum value of the logarithmic likelihood function; l
d. Akaike information criterion[12,14,15] ; AIC, defined by
   AIC = $-2\times l+2\times($ number of free parameters $)$
e. Sum of estimator y_i ; S_y
f. Upper and lower side of deviation between y_i and estimator $\hat{y}_i$; δ_+, δ_-
   $\delta_+=\max\ (y_i-\hat{y}_i)$ $[i=1\cdots n]$
   $\delta_-=\max\ (\hat{y}_i-y_i)$ $[i=1\cdots n]$

We should choose the better analysis method by using these indexes and judge the best fitting function with due regard to the conformity between the function and the phenomenon, for example,

a. contradictions in the range of variables

b. geometrical characteristics of the fitting function

4. CALCULATION AND RESULTS

(1) Method of calculation

As it is difficult to compare the results from deferent data, we use the data after M.Kobayashi which are shown in Tab.1. The fitting function, the form of $Y=g(X)$, to be investigate are as follows:

Ⅰ. $g_1(X) = a X + b$

Ⅱ. $g_2(X) = a \ln X + b$

Ⅲ. $g_3(X) = a X^b$

Ⅳ. $g_4(X) = a X^b + c$

where $X=x/N$ is the ratio of destroyed houses, $Y=y/N$ is the ratio of serious fires outbreak and a, b and c are the parameters to be searched. And x is the number of destroyed houses, N is the number of all houses, y is the number of serious fires outbreak. The sample data points are scattered in Fig.2. We use the function $Y=g(X)$ for the method of least squares and use the function $y=N \cdot g(X)$ for the Awaya method in view of property of data. The fitting functions given by the form of g(X) are compared. Furthermore we consider the case of the data with the Poisson errors in ordinate and abscissa, i.e. two-dimensional curve fitting.

(2) Results and discussion

The eight functions are obtained from the calculations by the two methods about four types of g(X) if we supposed the data with the errors in ordinate. The results are shown in Tab.2. The indexes which play important roles are the maximum value of the logarithmic likelihood function and the value of AIC from standpoint of statistical model and other indexes are used only for the information of tendency. The model whose value of the logarithmic likelihood function is greater is the better one. The model whose value of AIC is less is the better one. It is reported that the significant difference is the value over one or two in view of entropy. About of all four functions, the maximum value of the logarithmic likelihood function given by Awaya method is greater than that of the method of least squares and the value of AIC given by Awaya method is less than that of the method of least squares. It became clear that the Awaya method is better than the method of least squares in this case. A result which supports the assumption of the new method was obtained. Next, it has been shown that the function of ⑧ is the best fitted in view of the maximum value of the logarithmic likelihood function and that the function of ⑥ is the best fitted from standpoint of the value of AIC. The functions of ②, ④, ⑥ and ⑧ are traced in Fig.2. Lastly, we applied the method of two-dimensional fitting. The results are shown in Tab.3. The similar values of parameters to Tab.2 have been obtainedand and these table could hardly be compared in this case. In the former studies, only an arbitrary function was often fitted by the unsuitable method. However, it is important to compare several functions to be fitted and to investigate not only correlation coefficient but also the other indexes, the maximum value of logarithmic likelihood function and the value of AIC.

Tab.1 Data of Kanto earthquake(1923)

Item / District	All houses	Destroyed houses	Serious fires*
1. Kojinmachi	15430	409	6
2. Nihonbashi	19425	82	0
3. Kyobashi	21551	86	8
4. Shiba	27755	415	6
5. Azabu	14382	352	1
6. Akasaka	11691	360	4
7. Yotsuya	11417	48	1
8. Hongo	21321	149	4
9. Shimoya	30239	733	7
10. Kanda	18430	1262	10
11. Asakusa	32051	2218	19
12. Koishikawa	23686	144	3
13. Ushigome	19073	247	4
14. Honjo	32948	4426	17
15. Hukagawa	25570	2064	9
Total	324969	12995	199

* excluding chemical fires

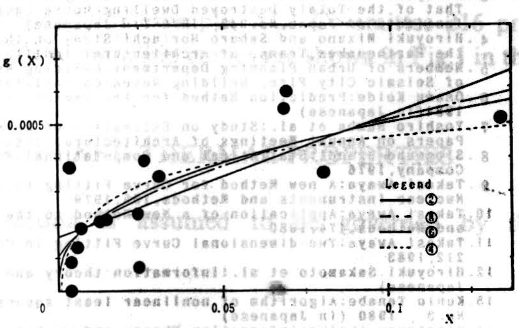


Fig.2 Scattered data and fitted curves

Tab.2 Results of the fitted function and these indexes

Fitted functions (Item)	R	S	$\hat{l}$	AIC	$\Sigma\hat{y}$	δ_-	δ_+
① $g_1(X) = 0.348 \times 10^{-2} X + 0.163 \times 10^{-3}$	0.865	107.39	-36.05	78.09	98.2	3.8	6.1
② $g_1(X) = 0.364 \times 10^{-2} X + 0.159 \times 10^{-3}$	0.864	109.72	-34.66	73.32	99.0	4.4	5.8
③ $g_2(X) = 0.114 \times 10^{-3} \ln X + 0.735 \times 10^{-3}$	0.880	96.30	-35.23	76.46	97.3	3.5	5.7
④ $g_2(X) = 0.102 \times 10^{-3} \ln X + 0.694 \times 10^{-3}$	0.882	98.47	-35.09	74.18	98.8	3.5	5.5
⑤ $g_3(X) = 0.132 \times 10^{-2} X^{0.417}$	0.887	90.04	-34.73	75.45	97.8	3.0	5.2
⑥ $g_3(X) = 0.125 \times 10^{-2} X^{0.396}$	0.888	89.37	-34.27	72.55	99.3	3.1	5.1
⑦ $g_4(X) = 0.150 \times 10^{-2} X^{0.514} + 0.524 \times 10^{-4}$	0.885	91.29	-34.83	77.66	98.3	3.0	5.1
⑧ $g_4(X) = 0.158 \times 10^{-2} X^{0.566} + 0.816 \times 10^{-4}$	0.885	91.71	-34.19	74.37	99.3	3.0	5.2

(Note) Least squares method; ①, ③, ⑤, ⑦. Awaya method; ②, ④, ⑥, ⑧

Tab.3 Results of two-dimensional fitting by Awaya method

Fitted functions (Item)	R	S	$\hat{l}$	AIC	$\Sigma\hat{y}$	δ_-	δ_+
⑨ $g_1(X) = 0.365 \times 10^{-2} X + 0.159 \times 10^{-3}$	0.864	109.90	-34.66	73.32	99.1	4.4	5.8
⑩ $g_2(X) = 0.104 \times 10^{-3} \ln X + 0.704 \times 10^{-3}$	0.881	97.59	-35.10	74.19	99.6	3.6	5.3
⑪ $g_3(X) = 0.125 \times 10^{-2} X^{0.398}$	0.888	89.47	-34.27	72.55	98.6	3.1	5.2
⑫ $g_4(X) = 0.154 \times 10^{-2} X^{0.522} + 0.763 \times 10^{-4}$	0.885	91.40	-34.19	74.37	98.9	3.0	5.3

5. CONCLUDING REMARKS

We have reviewed former studies on the number of seismic fire and pointed out the problems theoretically. Then we have applied the new method and obtained some findings. A summary of our conclusion is written as follows:

1) It has been shown by the theoretical consideration that the usual least squares curve fitting method is not suitable to the non-negative discrete data.
2) The new method proposed by Awaya has been tried to analyze the data of the number on seismic fire, i.e. an example of Kanto earthquake(1923). Then it has been proved that the Awaya method is more suitable than the method of least squares .
3) It has been shown by AIC that the function of ⑥ is the best fitted .
4) More theoretically we should apply the method of two-dimensional fitting. However similar values of parameters have been obtained in this case.
5) The former procedure to get the function is also not suitable. It is important to compare several functions to be fitted by using not only correlation coefficient but also the other indexes, the maximum value of logarithmic likelihood function and the value of AIC. This is the methodology which can give the most appropriate formula in view of statistical model.

Since this study is the basic one laying stress on the methodology, we should make these fruitful conclusions apply the more practical one.

ACKNOWLEDGMENT

We wish to thank Dr.Awaya, Prof. of Aoyama Gakuin Univ., for his helpful advice and for providing the computer programs. A part of this study is supported by scientific research fund from the Ministry of Education in Japan.

REFERENCES

1 .Tokyo Metropolis:Report of the Predictive Damage of Fire Outbreaks following large Earthquake in Tokyo,Tokyo Metropolis, 1976 (in Japanese)
2 .Masami Kobayashi:Analysis of Fire Outbreaks in the 1923 Kanto Earthquake,Trans. of Architectural Institute of Japan,No.337,1984 (in Japanese)
3 .Hiroyuki Mizuno and Saburo Horiuchi:Study on the Relationship between the Ratio of the Fires and That of the Totaly Destroyed Dwelling House Caused by the Earthquakes,Trans. of Architectural Institute of Japan,No.247, 1976 (in Japanese)
4 .Hiroyuki Mizuno and Saburo Horiuchi:Study on the Prediction of the Numbers of the Fires Caused by the Earthquakes,Trans. of Architectural Institute of Japan, No.250,1976 (in Japanese)
5 .Members of Urban Planning Department,Building Reserch Institute: Prediction of Breakout and spread of Seismic City Fire, Building Research Institute, Ministry of Construction, Japan,1983(in Japanese)
6 .Osamu Koide:Prediction Method for the damage of the Seismic Fire, Dissertation,Univ. of Tokyo, 1984(in Japanese)
7 .Yoshiro Namba et al.:Study on Estimation of Seismic Fire Break-out Rate, Summaries of Technical Papers on Annual Meetings of Architectural Institute of Japan,1984 (in Japanese)
8 .Siegmund Brandt:Statistical and Computational Methods in Data Analysis, North-Holland Publishing Company,1976
9 .Takasi Awaya:A new Method for Curve Fitting to the Data with Low Statistics not Using the χ^2-Method, Nuclear Instruments and Methods,165,1979
10.Takasi Awaya:Application of a New Method to the Analysis of Radioactive Decays,Nuclear Instruments and Methods,174,1980
11.Takasi Awaya:Two-dimensional Curve Fitting in Counting Experiments,Nuclear Instruments and Methods, 212,1983
12.Hiroyuki Sakamoto et al.:Information theory and Statistics , Kyoritsu Publishing Company, 1983 (in Japanese)
13.Kunio Tanabe:Algorithm of nonlinear least squares,Japanese Journal of Applied Statistics , Vol.9, No.3 , 1980 (in Japanese)
14.Hirotsugu Akaike:Information Theory and an Extention of Maximum Likelihood Principle, 2nd Inter.Symp. on Information Theory,Akademiai Kiado, Budapest,1973
15.Hirotsugu Akaike:On Entropy maximization Principle, Application Statistics (Krishnaia ed.), North-Holland Publishing Company,1977